All traces are ellipses

\[ a = b = c \Rightarrow \text{sphere} \]

Ellipsoid

Horizontal traces are ellipses
Vertical traces are hyperbolas
Axis of symmetry corresponds to the variable with negative coefficient

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]
12.6 Quadric Surfaces

Hyperboloid (two sheets)

- Vertical traces are hyperbolas
- Horizontal traces are ellipses
- $z = k, |k| > c$
- Axis of symmetry corresponds to the variable with positive coefficient

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Cone (elliptic)

- Horizontal traces are ellipses
- Vertical traces are hyperbolas
- $k \neq 0 \Rightarrow x = k \Rightarrow hyperbolas$
- $y = k \Rightarrow hyperbolas$
- $k = 0 \Rightarrow lines$
- Axis of symmetry corresponds to the variable on the left side

$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$
12.6 Quadric Surfaces

Horizontal traces are ellipses
Vertical traces are parabolas
Axis of symmetry corresponds to the variable raised to the first power

\[ \frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2} \]

Horizontal traces are hyperbolas
Vertical traces are parabolas

\[ \frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2} \]
### Elliptic cylinder

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

### Hyperbolic cylinder

\[ \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \]
21–28 Match the equation with its graph (labeled I–VIII). Give reasons for your choices.

21. \( x^2 + 4y^2 + 9z^2 = 1 \) \( \text{VII} \)
22. \( 9x^2 + 4y^2 + z^2 = 1 \) \( \text{IV} \)
23. \( x^2 - y^2 + z^2 = 1 \) \( \text{II} \)
24. \( -x^2 + y^2 - z^2 = 1 \) \( \text{III} \)
25. \( y = 2x^2 + z^2 \) \( \text{VI} \)
26. \( y^2 = x^2 + 2z^2 \) \( \text{I} \)
27. \( x^2 + 2z^2 = 1 \) \( \text{VIII} \)
28. \( y = x^2 - z^2 \) \( \text{V} \)
Reduce the equation to one of the standard forms, classify the surface, and sketch it.

31. \( x = 2y^2 + 3z^2 \)  
32. \( 4x - y^2 + 4z^2 = 0 \)
33. \( 4x^2 + y^2 + 4z^2 - 4y - 24z + 36 = 0 \)
34. \( 4y^2 + z^2 - x - 16y - 4z + 20 = 0 \)

31. \( x = \frac{y^2}{2} + \frac{z^2}{3} \)  or  \( \frac{x}{6} = \frac{y^2}{3} + \frac{z^2}{2} \)  
   elliptic paraboloid  
   vertex: \((0,0,0)\)  
   opens in the positive \( x \)-direction

32. \( x = \frac{y^2}{4} - \frac{z^2}{1} \)  
   hyperbolic paraboloid

Reduce the equation to one of the standard forms, classify the surface, and sketch it.

31. \( x = 2y^2 + 3z^2 \)  
32. \( 4x - y^2 + 4z^2 = 0 \)
33. \( 4x^2 + y^2 + 4z^2 - 4y - 24z + 36 = 0 \)
34. \( 4y^2 + z^2 - x - 16y - 4z + 20 = 0 \)

33. \( 4x^2 + y^2 - 4y + \frac{4}{4} + 4z^2 - 24z + \frac{36}{4} = -36 + \frac{4}{4} + \frac{36}{4} \)
   \( 4x^2 + \left( y^2 - 4y + \frac{4}{4} \right) + 4 \left( z^2 - 6z + \frac{9}{4} \right) = -36 + \frac{4}{4} + \frac{36}{4} \)
   \( 4x^2 + \left( y - 2 \right)^2 + 4 \left( z - 3 \right)^2 = 4 \)

\( \frac{x^2}{1} + \frac{(y-2)^2}{4} + \frac{(z-3)^2}{1} = 1 \)  
   ellipsoid  
   center: \((0,2,3)\)
Reduce the equation to one of the standard forms, classify
the surface, and sketch it.

31. \( x = 2y^2 + 3z^2 \)

32. \( 4x - y^2 + 4z^2 = 0 \)

33. \( 4x^2 + y^2 + 4z^2 - 4y - 24z + 36 = 0 \)

34. \( 4y^2 + z^2 - x - 16y - 4z + 20 = 0 \)

34. \( 4y^2 - 16y + 16z^2 - 4z + 4 = -20 + x + 16 + 4 \)

\[ 4(y^2 - 4y + 4) + (z^2 - 4z + 4) = -20 + x + 16 + 4 \]

\[ 4(y - 2)^2 + (z - 2)^2 = x \]

\[ \frac{x}{4} = \frac{(y - 2)^2}{1} + \frac{(z - 2)^2}{4} \]

elliptic paraboloid

vertex: \((0, 2, 2)\)

Summary:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 )</td>
<td>Ellipsoid</td>
</tr>
<tr>
<td>( \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 )</td>
<td>Hyperboloid of one sheet</td>
</tr>
<tr>
<td>( \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 )</td>
<td>Hyperboloid of two sheets</td>
</tr>
<tr>
<td>( \frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2} )</td>
<td>Cone (elliptic)</td>
</tr>
<tr>
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<td>Paraboloid (elliptic)</td>
</tr>
<tr>
<td>( \frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2} )</td>
<td>Paraboloid (hyperbolic)</td>
</tr>
</tbody>
</table>

all variables present
all variables squared
all variables present
one variable not squared

one variable not present \(\Rightarrow\) cylinder opening in the direction of the missing variable

\( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) Elliptic cylinder

\( \frac{x^2}{a^2} - \frac{z^2}{b^2} = 1 \) Hyperbolic cylinder

\( z = ax^2 \) Parabolic cylinder