12.4 Cross Product

The **cross product** of two vectors is a _____ with the special quality of being ___________ to both original vectors.

The cross product yields a ____ in contrast to the dot product that yields a ____________

The **cross product** of \( \mathbf{u} = \langle u_1, u_2, u_3 \rangle \) and \( \mathbf{v} = \langle v_1, v_2, v_3 \rangle \) is

\[
\mathbf{u} \times \mathbf{v} = \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle
\]

The definition __________________________________

(The cross product is ____ defined for two-dimensional vectors.)

Instead of memorizing what gets multiplied by what, there is a convenient way to calculate \( \mathbf{u} \times \mathbf{v} \) using the _________ form with ________________.

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### Determinant

\[
\begin{vmatrix} a & b \\ c & d \end{vmatrix} =
\]

\[
\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} =
\]

\[
\begin{vmatrix} -6 & 2 \\ -9 & 3 \end{vmatrix} =
\]

\[
\begin{vmatrix} 0 & 3 \\ -1 & 99 \end{vmatrix} =
\]
3X3 Determinants

Reduces to finding __ 2×2 determinants using cofactor expansion on the _______.

Take each entry in the first row, we will multiply each of these entries by a 2×2 determinant.

The 2×2 determinants are found by __________ that entry's column and row.

One last thing is to _________________.

\[
\begin{vmatrix}
1 & -3 & 2 \\
-1 & 9 & 4 \\
-5 & 3 & 1 \\
\end{vmatrix} = \]

\[
\begin{vmatrix}
1 & 6 & -2 \\
3 & -1 & 3 \\
4 & 5 & 2 \\
\end{vmatrix} = \]

3×3 Shortcut

\[
\begin{vmatrix}
1 & 6 & -2 \\
3 & -1 & 3 \\
4 & 5 & 2 \\
\end{vmatrix} = \left( \text{sum of forward diagonal products} \right) - \left( \text{sum of backward diagonal products} \right)
\]

How to find the cross product using determinants

\[
\mathbf{u} \times \mathbf{v} = \begin{vmatrix}
i & j & k \\
u_1 & u_2 & u_3 \\
v_1 & v_2 & v_3 \\
\end{vmatrix} = 
\]

\[
\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2)i - (u_1v_3 - u_3v_1)j + (u_1v_2 - u_2v_1)k 
\]

\[
\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1) 
\]

Let \( \mathbf{u} = \langle 1, -2, 1 \rangle \) and \( \mathbf{v} = \langle 3, 1, -2 \rangle \) Find \( \mathbf{u} \times \mathbf{v} \).

\[
\begin{vmatrix}
i & j & k \\
1 & -2 & 1 \\
3 & 1 & -2 \\
\end{vmatrix} = 
\]
Let \( \mathbf{u} = \langle 1, 1, 1 \rangle \) and \( \mathbf{v} = \langle 2, 1, -1 \rangle \). Find \( \mathbf{u} \times \mathbf{v} \) and show that it is orthogonal to both \( \mathbf{u} \) and \( \mathbf{v} \).

Algebraic Properties of the cross product:

Let \( \mathbf{u}, \mathbf{v}, \) and \( \mathbf{w} \) be vectors and let \( c \) be a scalar.

1. \( \mathbf{u} \times \mathbf{v} = \)
2. \( \mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \)
3. \( c(\mathbf{u} \times \mathbf{v}) = \)
4. \( 0 \times \mathbf{v} = \)
5. \( \mathbf{v} \times \mathbf{v} = \)
6. \( \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \)
7. \( \mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \)
**Right-hand rule**

Place your 4 fingers in the direction of the ________, ___ them in the direction of the ____________, Your _____ will point in the direction of the cross product

\[ \mathbf{u} \times \mathbf{v} = - (\mathbf{v} \times \mathbf{u}) \]  
(by switching the order, you get a vector __________________)

Geometric Properties of the cross product:

Let \( \mathbf{u} \) and \( \mathbf{v} \) be nonzero vectors and let \( \theta \) be the angle between \( \mathbf{u} \) and \( \mathbf{v} \).

1. \( \mathbf{u} \times \mathbf{v} \) is ____________ to both \( \mathbf{u} \) and \( \mathbf{v} \).

2. \( |\mathbf{u} \times \mathbf{v}| = \)

3. \( \mathbf{u} \times \mathbf{v} = 0 \) if and only if

4. \( |\mathbf{u} \times \mathbf{v}| = \)

5. \( \frac{1}{2} |\mathbf{u} \times \mathbf{v}| = \)
A nice online java applet for the cross product can be found here:

http://www.phy.syr.edu/courses/java-suite/crosspro.html

Volume of the parallelepiped determined by the vectors $\mathbf{a}, \mathbf{b},$ and $\mathbf{c}$.

Area of the base = 

Height = 

Volume = 

$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ is called the ______________

The vectors are in the same plane (______) if the scalar triple product ____.

The scalar triple product can be written as a determinant:
Let \( \mathbf{u} = \langle 2,0,1 \rangle \), \( \mathbf{v} = \langle 1,1,1 \rangle \) and \( \mathbf{w} = \langle 0,2,2 \rangle \). Find \( \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) \).

In physics, the cross product is used to measure ______.

Consider a force \( \mathbf{F} \) acting on a rigid body at a point given by a position vector \( \mathbf{r} \).

The ______ (\( \tau \)) measures the tendency of the body to ______ about the origin (point \( P \))

\[
\tau =
\]

\[
|\tau| =
\]

\( \theta \) is the angle between the______ and ______ vectors (\( \theta \) is the angle between the ______ and ______ vectors)