A projectile is launched, we want to find the parametric equations for the path.

- the projectile has mass $m$
- assume that gravity is the only force acting on the projectile after it is launched (neglect air resistance)
- Force due to gravity: $\mathbf{F} = (-mg)\mathbf{j}$
- Newton's Second Law: $\mathbf{F} = ma \Rightarrow (-mg)\mathbf{j} = ma$

$$\mathbf{a} = (-g)\mathbf{j}$$

$$\mathbf{v} = \int \mathbf{a}(t)\,dt$$

$$\mathbf{r} = \int \mathbf{v}(t)\,dt$$

$$\mathbf{v}(t) =$$

$$\mathbf{r}(t) =$$
\[ \mathbf{a} = (-g) \mathbf{j} \]
\[ \mathbf{v}(t) = -gt \mathbf{j} + \mathbf{v}_0 \]
\[ \mathbf{r}(t) = -\frac{1}{2} gt^2 \mathbf{j} + \mathbf{v}_0 t + \mathbf{r}_0 \]

Often you are given an initial height, an initial speed, and the angle \( \theta \) at which the projectile is launched.

\[ \mathbf{v}_0 = \langle |\mathbf{v}_0| \cos \theta, |\mathbf{v}_0| \sin \theta \rangle \]
\[ \mathbf{r}_0 = \langle 0, h \rangle \]
\[ \mathbf{a} = (-g) \mathbf{j} \quad \mathbf{a}(t) = \langle 0, -g \rangle \]
\[ \mathbf{v}(t) = \langle 0, -gt \rangle + \langle |\mathbf{v}_0| \cos \theta, |\mathbf{v}_0| \sin \theta \rangle \]
\[ \mathbf{r}(t) = \langle 0, -\frac{1}{2} gt^2 \rangle + \langle |\mathbf{v}_0| \cos \theta, |\mathbf{v}_0| \sin \theta \rangle t + \langle 0, h \rangle \]
A baseball is hit 3 feet above the ground level at 100 feet per second and at an angle of 45° with respect to the ground. Find the maximum height reached by the baseball. Will it clear a 10 foot fence located 300 feet from home plate?

\[ h = \theta = \]  
\[ |v_0| = g = \]

\[ r(t) = \left( (|v_0|\cos \theta) t, h + (|v_0|\sin \theta) t - \frac{1}{2}gt^2 \right) = \left( (100\cos 45^\circ)t, 3 + (100\sin 45^\circ)t - \frac{1}{2}(32)t^2 \right) \]

\[ r(t) = \]

The maximum height occurs when the \underline{__________} of the velocity vector is \underline{_____}.

\[ v(t) = \langle 50\sqrt{2}, 50\sqrt{2} - 32t \rangle \Rightarrow 50\sqrt{2} - 32t = 0 \Rightarrow t = \frac{25\sqrt{2}}{16} \approx 2.21 \text{ sec.} \]

The maximum height \(= 3 + 50\sqrt{2} \left( \frac{25\sqrt{2}}{16} \right) - 16 \left( \frac{25\sqrt{2}}{16} \right)^2 \)

The ball is 300 ft. from home plate when the \underline{__________} of the \underline{______} vector is 300.

\[ \Rightarrow 50\sqrt{2}t = 300 \]
\[ t = \frac{6}{\sqrt{2}} = 3\sqrt{2} \approx 4.24 \text{ sec.} \]

Plug this time into the \underline{__________} of the position vector to find the height of the ball.

The height of the ball at the fence \(= 3 + 50\sqrt{2} \left( 3\sqrt{2} \right) - 16 \left( 3\sqrt{2} \right)^2 \)
Height, Flight Time, and Range for Ideal Projectile Motion

For ideal projectile motion when an object is launched from the origin over a horizontal surface with initial speed $v_0$ and launch angle $\alpha$:

Maximum height: $y_{\text{max}} = \frac{(v_0 \sin \alpha)^2}{2g}$

Flight time: $t = \frac{2v_0 \sin \alpha}{g}$

Range: $R = \frac{v_0^2}{g} \sin 2\alpha$. 
23. **Firing golf balls**  A spring gun at ground level fires a golf ball at an angle of 45°. The ball lands 10 m away.

   a. What was the ball’s initial speed?

   b. For the same initial speed, find the two firing angles that make the range 6 m.
\[ a = \]

\[ v(t) = \]

\[ r(t) = \]

2. In Citizens Bank Park, where the Philadelphia Phillies play baseball, the right-field fence is 330 feet from home plate (where the batter stands when he hits), and the fence is about 13 feet high. First-baseman Ryan Howard hits a home run over the right-field fence that starts out 3 feet above home plate with horizontal velocity 66 ft/sec toward the wall and initial upward velocity 84 ft/sec. By how many feet does the ball clear the top of the fence? (Assume the acceleration due to gravity is 32 ft/sec\(^2\) and ignore wind resistance.)