14.4 Chain Rule

\[ z = f(x, y) \]

\[ x = g(t) \]
\[ y = h(t) \]

\[ \frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} \]

Let \( z = e^{xy} \cos(x^2) \) with \( x = \sqrt{t} \) and \( y = \ln t \). Use the chain rule to find \( \frac{dz}{dt} \) at \((1, 0)\).
\[ z = f(x, y) \]
\[ x = g(s, t) \]
\[ y = h(s, t) \]

\[
\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s}
\]

\[
\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t}
\]
Implicit Differentiation

\[ z = f(x, y) \]

\( y \) defined implicitly as a function of \( x \)

\[ y = g(x) \]

\[ z = f(x, g(x)) \]

Set the function equal to 0.

\[ w = F(x, y) \]

\[ \frac{dy}{dx} = - \frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = - \frac{F_x}{F_y} \]
**Implicit Differentiation**

\[ w = f(x, y, z) \]

\[ \frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} = -\frac{F_x}{F_z} \]

\[ z = g(x, y) \]

\[ \frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} = -\frac{F_y}{F_z} \]

Set the function equal to 0.

\[ p = F(x, y, z) \]