15.1 Intro. to Double Integrals

Single variable integral: area under the graph of the function and above the x-axis found by using the area of infinitely many rectangles.

\[ A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x \quad A = \int_{a}^{b} f(x) \, dx \]

Double variable integral: volume under the graph of the function (surface) and above the xy-plane found by using the volume of infinitely many rectangular prisms.
15.1 Intro. To Double Integrals


d = y_m

\(\sum \sum \)

\[V \approx \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i^*, y_j^*) \Delta A\]
Approximate the volume $V$ of the solid lying under the graph of the elliptic paraboloid $z = 8 - 2x^2 - y^2$ and above the rectangle $R = \{(x,y) \mid 0 \leq x \leq 1, 0 \leq y \leq 2\}$.

Using the partition $P$ of $R$ that is obtained by dividing $R$ into four rectangles with the lines $x = \frac{1}{2}$ and $y = 1$ and use the evaluation point $(x^*_i, y^*_j)$ to be the upper right hand corner of each subrectangle.

The approximations get better and better as $m$ and $n$ increase.

$$ V = \lim_{m,n \to \infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x^*_i, y^*_j) \Delta A = \iint_R f(x, y) \, dA $$

**Fubini’s Theorem for Rectangular Regions**

Let $f$ be continuous over the rectangle $R = \{(x,y) \mid a \leq x \leq b, c \leq y \leq d\}$, then

$$ \iint_R f(x, y) \, dA = $$
15.1 Iterated Integrals

\[
\iiint_R f(x, y) \, dA = \int_a^b \left[ \int_x^c f(x, y) \, dx \right] dy = \int_a^b \left[ \int_y^c f(x, y) \, dy \right] dx
\]

Integrate w.r.t. \( \_ \) first treating \( \_ \) as a constant
Work \( \_ \)

This inside integral can be thought of as a function of \( y \), call it \( A(y) \)

This inside integral can be thought of as a function of \( x \), call it \( A(x) \)

Find the volume \( V \) of the solid lying under the graph of the elliptic paraboloid \( z = 8 - 2x^2 - y^2 \) and above the rectangle \( R = \left\{ (x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 2 \right\} \)
Evaluate
\[ \int_{0}^{1} \int_{2}^{3} \frac{1}{(x + 4y)^3} \, dx \, dy \]