Change of variable

\[ T : S \rightarrow R \]
\[(x, y) = (f(u,v), g(u,v))\]

Jacobian

\[ J(u, v) = \begin{vmatrix}
\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\
\frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}
\end{vmatrix} = \frac{\partial x}{\partial u} \cdot \frac{\partial y}{\partial v} - \frac{\partial x}{\partial v} \cdot \frac{\partial y}{\partial u} \]

This is covered in section 15.8

Element of area becomes
\[ dxdy \]
\[ |J(u, v)| dudv \]

Provided:
\( f \) and \( g \) have continuous first partial derivatives on \( S \)

\( T \) is one-to-one

\( R \) and \( S \) consist of a piecewise smooth simple closed curve and its interior

\[ J \neq 0 \]

\[ \iint_S F(x, y) dA = \iint_R F(f(u,v), g(u,v)) |J(u, v)| dudv \]
Conversion of Rectangular into Polar:

\[ x = r \cos \theta \quad \frac{\partial x}{\partial r} = \quad \frac{\partial x}{\partial \theta} = \]
\[ y = r \sin \theta \quad \frac{\partial y}{\partial r} = \quad \frac{\partial y}{\partial \theta} = \]

\[ J(r, \theta) = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} \]

\[ dx \, dy = \left| J(r, \theta) \right| \, dr \, d\theta = \]
or \[ dy \, dx \]
15.4 Double Integrals in Polar Coordinates

Change of variables

\[ x = f(u, v), \quad y = g(u, v) \]

Jacobian

\[ \frac{\partial (x, y)}{\partial (u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} \]

\[
\iiint_{R} F(x, y) \, dA = \iiint_{S} F(f(u, v), g(u, v)) \left| \frac{\partial (x, y)}{\partial (u, v)} \right| \, dudv
\]

absolute value of the Jacobian

Change of variables into Polar Coordinates

\[ x = r \cos \theta, \quad y = r \sin \theta \]

Jacobian

\[
\frac{\partial (x, y)}{\partial (r, \theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix} = r \cos^2 \theta + r \sin^2 \theta = r
\]

\[
\iiint_{R} F(x, y) \, dA = \iiint_{R} F(r \cos \theta, r \sin \theta) \, r \, dr \, d\theta
\]
Area of a region in Polar Coordinates

Area of Region \( R = \iint_R dA = \iint_R r\,dr\,d\theta \) in polar coordinates

Find the area enclosed by the graph of

\[ r = 2\sin 2\theta \quad 0 \leq \theta \leq \frac{\pi}{2} \]
Evaluate the given integral by converting to polar coordinates.

\[
\int_{0}^{1} \int_{0}^{\sqrt{1-x^2}} e^{x^2 + y^2} \, dy \, dx
\]

Find the region:

\[
0 \leq y \leq \sqrt{1-x^2}
\]

\[
0 \leq x \leq 1
\]
\[ \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{0} \frac{2}{1 + \sqrt{x^2 + y^2}} \, dy \, dx \]

Find the region:

\[-\sqrt{1-x^2} \leq y \leq 0\]

\[-1 \leq x \leq 1\]
Evaluate the given integral by converting to polar coordinates:

\[ \int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} \, dy \, dx \]

Find the region:

\[ 0 \leq y \leq \sqrt{2x-x^2} \]
\[ 0 \leq x \leq 2 \]