Chapter 2: The Basic Concepts of Set Theory

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Section 2-1
Symbols and Terminology

Symbols and Terminology

- Designating Sets
- Sets of Numbers and Cardinality
- Finite and Infinite Sets
- Equality of Sets
A set is a collection of objects. The objects belonging to the set are called the elements, or members of the set.

Sets are designated using:

1) word description,
2) the listing method, and
3) set-builder notation.

Word description
The set of even counting numbers less than 10

The listing method
{2, 4, 6, 8}

Set-builder notation
{x | x is an even counting number less than 10}
Designating Sets

Sets are commonly given names (capital letters).
\[ A = \{1, 2, 3, 4\} \]

The set containing no elements is called the **empty set** (*null set*) and denoted by \( \{\} \) or \( \emptyset \).

To show 2 is an element of set \( A \) use the symbol \( \in \).
\[ 2 \in \{1, 2, 3, 4\} \]
\[ a \notin \{1, 2, 3, 4\} \]

Example: Listing Elements of Sets

Give a complete listing of all of the elements of the set \( \{x | x \text{ is a natural number between 3 and 8}\} \)

**Solution**
\[ \{4, 5, 6, 7\} \]
Sets of Numbers

Natural (counting)  \{1, 2, 3, 4, \ldots\}
Whole numbers \{0, 1, 2, 3, 4, \ldots\}
Integers \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}

Rational numbers \left\{ \frac{p}{q} \mid p \text{ and } q \text{ are integers, with } q \neq 0 \right\}

May be written as a terminating decimal, like 0.25, or a repeating decimal like 0.333…

Irrational \{x \mid x \text{ is not expressible as a quotient of integers}\}

Decimal representations never terminate and never repeat.

Real numbers \{x \mid x \text{ can be expressed as a decimal}\}

Cardinality

The number of elements in a set is called the cardinal number, or cardinality of the set.

The symbol \( n(A) \), read “\( n \) of \( A \),” represents the cardinal number of set \( A \).
Example: Cardinality

Find the cardinal number of each set.

a) \( K = \{ a, l, g, e, b, r \} \)
b) \( M = \{ 2 \} \)
c) \( \emptyset \)

Solution

a) \( n(K) = 6 \)
b) \( n(M) = 1 \)
c) \( n(\emptyset) = 0 \)

Finite and Infinite Sets

If the cardinal number of a set is a particular whole number, we call that set a finite set.

Whenever a set is so large that its cardinal number is not found among the whole numbers, we call that set an infinite set.
Example: Infinite Set

The odd counting numbers are an infinite set.

Word description
The set of all odd counting numbers

Listing method
{1, 3, 5, 7, 9, …}

Set-builder notation
{x | x is an odd counting number}

Equality of Sets

Set $A$ is equal to set $B$ provided the following two conditions are met:

1. Every element of $A$ is an element of $B$, and
2. Every element of $B$ is an element of $A$. 
Example: Equality of Sets

State whether the sets in each pair are equal.

a) \{a, b, c, d\} and \{a, c, d, b\}

b) \{2, 4, 6\} and \{x| x is an even number\}

Solution

a) Yes, order of elements does not matter

b) No, \{2, 4, 6\} does not represent all the even numbers.