"Del", \( \nabla \) - A defined operator

\[
\nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)
\]

The **gradient** of a function (at a point) is a vector that points in the direction in which the function increases most rapidly.

A **vector field** is a vector function that can be thought of as a velocity field of a fluid. At each point it assigns a vector that represents the velocity of a particle at that point.

The **flux** of a vector field is the volume of fluid flowing through an element of surface area per unit time.

The **divergence** of a vector field is the flux per unit volume.

The divergence of a vector field is a number that can be thought of as a measure of the rate of change of the density of the fluid at a point.

The **curl** of a vector field measures the tendency of the vector field to rotate about a point.

The curl of a vector field at a point is a vector that points in the direction of the axis of rotation and has magnitude represents the speed of the rotation.
**Vector Field**

\[ \mathbf{F} = \langle P(x, y, z), Q(x, y, z), R(x, y, z) \rangle \]

**Scalar Function**

\[ f(x, y, z) \]

**Gradient**

\[ \nabla f = \langle f_x, f_y, f_z \rangle \]

**Divergence**

\[ \nabla \cdot \mathbf{F} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot \langle P, Q, R \rangle = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = P_x + Q_y + R_z \]

**Curl**

\[ \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \langle R_y - Q_z, - (R_z - P_x), Q_x - P_y \rangle \]

For the given function \( \mathbf{F}(x, y, z) = \langle xe^{-z}, 4yz^2, 3ye^{-z} \rangle \):

\[ \text{div} \mathbf{F} = \nabla \cdot \mathbf{F} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot \langle xe^{-z}, 4yz^2, 3ye^{-z} \rangle = e^{-z} + 4z^2 - 3ye^{-z} \]

\[ \text{curl} \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xe^{-z} & 4yz^2 & 3ye^{-z} \end{vmatrix} = \langle 3e^{-z} - 8yz^2, -(0 - (-xe^{-z})), 0 \rangle \]

\[ \langle 3e^{-z} - 8yz^2, -xe^{-z}, 0 \rangle \]
\[ \text{grad (scalar function)} = \text{Vector Field} \]
\[ \text{div (Vector Field)} = \text{scalar function} \]
\[ \text{curl (Vector Field)} = \text{Vector Field} \]

Which of the 9 ways to combine grad, div and curl by taking one of each. Which of these combinations make sense?

\[ \checkmark \text{grad (div (F))} \quad \text{div (grad (f))} \quad \checkmark \text{curl (grad (f))} \quad 0 \text{ vector} \]

\[ \checkmark \text{grad (curl (F))} \quad \text{div (curl (F))} \quad \checkmark \text{curl (curl (F))} \quad 0 \text{ scalar} \]

2 of the above are always zero.

Verify the given identity. Assume continuity of all partial derivatives.
\[ \text{curl (grad (f))} = 0. \]
\[ \text{grad (f)} = \{f_x, f_y, f_z\} \]
\[ \text{curl (grad (f))} = \nabla \times \nabla f = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix} \]
\[ \nabla \times \nabla f = \{f_{xy} - f_{yz}, (f_{xz} - f_{yz}), f_{xz} - f_{yz}\} = 0 \]
since mixed partial derivatives are equal.

Verify the given identity. Assume continuity of all partial derivatives.
\[ \text{div (curl (F))} = 0. \]
\[ \text{Let } F = \{P(x,y,z), Q(x,y,z), R(x,y,z)\} \]
\[ \text{curl (F)} = \{R_y - Q_z, P_z - R_x, Q_x - P_y\} \]
\[ \text{div (curl (F))} = \{R_y - Q_z + R_z - R_x + Q_x - P_y\} \]
\[ \text{div (curl (F))} = -P_x + P_x + Q_y - Q_y + R_z - R_z + 0 \]
\[ \text{div (curl (F))} = 0. \]