13.1 Separable Partial Differential Equations

Linear Homogeneous Second-Order Partial Differential Equation (general form)

Solution: \( u(x, y) \)

\[
A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D \frac{\partial u}{\partial x} + E \frac{\partial u}{\partial y} + Fu = 0
\]

A, B, C, D, E, and F are real constants

alternate notation

\[
Au_{xx} + Bu_{xy} + Cu_{yy} + Du_{x} + Eu_{y} + Fu = 0
\]

linear:

homogeneous:

second – order:

partial diff. eq. :
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Classification:

\[ A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D \frac{\partial u}{\partial x} + E \frac{\partial u}{\partial y} + Fu = 0 \]

1. \( B^2 - 4AC > 0 \) \( \Rightarrow \) 

2. \( B^2 - 4AC < 0 \) \( \Rightarrow \) 

3. \( B^2 - 4AC = 0 \) \( \Rightarrow \)

_____ Equation

Solution: \( u(x,t) \)

\( k \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \) \( k > 0 \)

\( A = k, B = 0, C = 0 \)

_____ Equation

Solution: \( u(x,t) \)

\( a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \)

\( A = a^2, B = 0, C = -1 \)

_____ Equation

Solution: \( u(x, y) \)

\( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \)

\( A = 1, B = 0, C = 1 \)
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Separation of Variables

want to find a particular solution of the form: ________________
this will sometimes reduce a linear PDE in two variables into _______

\[ u(x, y) = \]
\[ \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial u}{\partial x} = \]
\[ \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial y} = \]

where the prime denotes ordinary differentiation
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example:

#9 \[ k \frac{\partial^2 u}{\partial x^2} - u = \frac{\partial u}{\partial t}, \quad k > 0 \]

\[ k \frac{\partial^2 u}{\partial x^2} - u = \frac{\partial u}{\partial t} \Rightarrow \]

or shorthand:

\[ u(x,t) = X(x)T(t) \]

\[ k \frac{\partial^2 u}{\partial x^2} = \]

\[ \frac{\partial u}{\partial t} = \]

\[ \text{indep. of } t \quad \text{indep. of } x \]

\[ \text{but } = \text{ to a } \quad \text{but } = \text{ to a } \]

\[ \text{function of } t \quad \text{function of } x \]

\[ \Rightarrow \text{both sides are indep. of ________} \]

or each side is equal to a ________

\[ = -\lambda \]

\[ \Rightarrow \text{and } \]

\[ \text{as a convenience we use the } \]

\[ \text{________________________} - \lambda \]
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Solve: $T' + \lambda T = 0$

Solve: $kX'' - X + \lambda X = 0$