Section 18.1 Contour Integrals

Let’s go back to:
Section ____________

Parametric Curve

\[ x = f(t), \ y = g(t) \]

Orientation of the curve

\[ \int_{C} f(z) \, dz = \]

for continuous \( f \) and smooth \( C \)
Evaluate
\[ \int_C z^2 \, dz \text{ where } C \text{ is defined by } z(t) = 3t + 2it, -2 \leq t \leq 2 \]

\[ f(z) = z^2 \text{ so } f(z(t)) = \]
\[ z(t) = 3t + 2it \Rightarrow z'(t) = \]
\[ dz = z'(t) \, dt \Rightarrow dz = \]

\[ \int_C z^2 \, dz = \int_{-2}^{2} f(z(t)) \, z'(t) \, dt = \]
\[ = \]
\[ = \]
\[ = \]

18.1 Contour Integrals
**Odds and ends from Section 18.1**

Complex functions can be thought of as ________________

Consider $f(z)$ as a ______ at the point $z$.

$$f(z) = u(x, y) + iv(x, y)$$

______________ representation for the path

of a particle in the flow $\rightarrow r(t) =$

has ________ vector $\mathbf{T} = r'(t) = x'(t) + iy'(t)$

that must coincide with the function

$$f(x(t) + iy(t)) = u(x(t), y(t)) + iv(x(t), y(t))$$

$$\Rightarrow x'(t) = \quad \text{and} \quad y'(t) = \quad \Rightarrow \frac{dx}{dt} = \quad \text{and} \quad \frac{dy}{dt} =$$

the solutions $x(t)$ and $y(t)$ are called the __________ of the flow

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Find the streamlines of the flow associated with $f(z) = iz$

$$f(z) =$$

$$f(z) =$$

$$u = \quad \text{and} \quad v =$$

$$\Rightarrow \frac{dx}{dt} = \quad \text{and} \quad \frac{dy}{dt} =$$

$$\frac{dy}{dx} = \quad \Rightarrow \frac{dy}{dx} = \quad \text{Sep. of Var.}$$

the streamlines are ______

centered at the origin
Continuing to view the complex function as a flow, we can now calculate ________ and ________.

$C$: positively oriented simple closed curve

The ________ around $C$ measures the tendency of the flow to rotate $C$

The ________ across $C$ is the difference between the rate at which fluid enters and the rate at which fluid leaves the region bounded by $C$.

Bounding Theorem

$f$ ________ on a _____ curve $C$ and ________

$\Rightarrow$ (Length of $C$)

for all $z$ on $C$