Section 18.4
Cauchy's Integral Formula

If a) \( f(z) \) is an ______ function in a ____________ domain \( D \)

b) \( C \) is a ___________ contour in \( D \) traversed ________

c) \( z_0 \) is any point ________,

then,

\[ f(z_0) = \]

or \[ \oint_C \frac{f(z)}{z-z_0} \, dz = \]

\[ \oint_C \frac{z^2 - 3z + 4i}{z + 2i} \, dz \quad C : |z| = 3 \]

Cauchy’s Integral Formula
\[ \oint_C \frac{f(z)}{z-z_0} \, dz = 2\pi if(z_0) \]

\( f(z) = \quad z_0 = \)

\( f(z_0) = \)

\( f(z_0) = \quad \Rightarrow f(z_0) = \)

\( 2\pi if(z_0) = \quad \Rightarrow 2\pi if(z_0) = \)
Cauchy's Integral Formula for Derivatives

If a) $f(z)$ is an _______ function in a ___________ domain $D$

b) $C$ is a ___________ contour in $D$ traversed ____________

c) $z_0$ is any point _______,

then,

$$f^{(n)}(z_0) =$$

or

$$\oint_{C} \frac{f(z)}{(z-z_0)^{n+1}} dz =$$

$$\oint_{C} \frac{1}{z^3(z-1)^2} dz \quad C : |z-2| = 5$$

$C_1 : |z| = \quad C_2 : |z-1| =$

= $\oint_{C} \frac{f(z)}{(z-z_0)^{n+1}} dz = \frac{2\pi i}{n!} f^{(n)}(z_0)$
\[
\oint_{C} \frac{3z+1}{z(z-2)^2} \, dz
\]

\[
= \quad 18.4 \ # \ 23
\]

**A Bounding Theorem**

If \( f(z) \) is continuous on a smooth curve \( C \) and if \( |f(z)| \leq M \) for all \( z \) on \( C \), then

\[
\left| \oint_{C} f(z) \, dz \right| \leq ML, \quad \text{where } L \text{ is the length of } C.
\]

**Cauchy’s Inequality**

If we take \( C \) to be the circle \( |z - z_0| = r \) and if \( |f(z)| \leq M \) for all \( z \) on \( C \), then

\[
|f^{(n)}(z_0)| \leq \frac{n!M}{r^n}
\]

**Liouville’s Theorem**

The only _______ ________ functions are __________.