Section 19.2 Taylor Series

A power series with a nonzero radius of convergence represents an analytic function (within its circle of convergence).

Now we want to consider the reverse process:

Given an __________ can we find its ______________________________.

Let 

\[ f(z) \\]

be ________

within a ________

and 

\[ z_0 \]

be a ________

for \( z_0 = 0 \)

it is called the

______________

\[ f \]

has the series representation

\[ f(z) = \]

valid for the largest ________

with center \( z_0 \) and ________

that lies ________ within \( D \)

for \( f \) centered at \( z_0 \)

Familiar functions and their Maclaurin series:

\[ \frac{1}{1-z} = 1 + z + z^2 + z^3 + \cdots = \sum_{n=0}^{\infty} z^n \]

\[ e^z = 1 + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} + \cdots = \sum_{n=0}^{\infty} \frac{z^n}{n!} \]

\[ \sin z = z - \frac{z^3}{6} + \frac{z^5}{120} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!} \]

\[ \cos z = 1 - \frac{z^2}{2} + \frac{z^4}{24} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!} \]
Expand in a Maclaurin series and find the radius of convergence.

\[ f(z) = \frac{z}{1+z^2} \]

\[ L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \]

\[ f(z) = \frac{z}{(1-z)^3} \]

The radius of convergence of a differentiated series is \[ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \] radius of convergence of the original series.

Expand in a Taylor series centered at \( z_0 \) and find the radius of convergence.

\[ f(z) = \frac{1}{3-z} \]

\[ z_0 = 2i \]

\[ f(z) = \frac{1}{3-z+2i} \]

\[ = \frac{1}{3} \left[ 1 + \left( \frac{z-2i}{3-2i} \right) + \left( \frac{z-2i}{3-2i} \right)^2 + \left( \frac{z-2i}{3-2i} \right)^3 + \cdots \right] \]

\[ = \sum_{n=0}^{\infty} \]

\[ L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \]

\[ R = \frac{1}{L} \Rightarrow \]
A point $z = z_0$ where a complex function fails to be analytic is called a __________ or __________ of the function.

$$f(z) = \frac{2z}{z^2 + 1}, \quad z_0 = \quad \text{are singularities of } f(z).$$

A point $z = z_0$ is called an ______________ of a function if there exists some "deleted" neighborhood or "punctured" open disk of $z_0$, $0 < |z - z_0| < R$ where the function is __________.

Both $i$ and $-i$ are isolated singularities of $f(z) = \frac{2z}{z^2 + 1}$ since $f$ is analytic on the deleted neighborhood and

__________________ is the distance from the center $z_0$ of the series to the nearest ______________ $z_*$.  

$$f(z) = \frac{1}{3-z}, \quad R =$$  

$$z_0 = 2i, \quad z_* =$$  

$R =$