There are 12 problems below, answer all of them. To answer circle the ENTIRE statement you deem correct in the problem concerned. No work required to be shown, no partial credit, 2 points removed for each wrong answer, but no credit or removal of same for any question left blank. No books, tables, notes, calculators, phones or electronic equipment allowed; one 3 inch by 5 inch handwritten card (both sides) allowed. Use the backs of the exam sheets and the extra sheets at the end for your computations and scratch work.

YOUR NAME (print please) 

YOUR PENN ID NUMBER

YOUR SIGNATURE

CIRCLE YOUR SECTION TIME: W 9AM  W 10AM  F 9AM  F 10AM

I) Let $\Omega$ be the region inside the disc of radius 2 centered at the origin and write $\gamma$ for the boundary of $\Omega$. Then the integral around $\gamma$ of the function: $f(z) = \frac{e^z}{2z^2 + 11z + 15}$, is equal to

a) 0 
b) $4\pi i$ 
c) $2\pi i$ 
d) $-2\pi i$ 
e) $-4\pi i$

II) Now let $\Omega$ be the disc of radius 4 centered at the origin and again write $\gamma$ for its boundary. Then the integral around $\gamma$ of the function: $f(z) = (z + \frac{1}{z})$, is equal to

a) 0 
b) $4\pi i$ 
c) $2\pi i$ 
d) $-2\pi i$ 
e) $-4\pi i$
III) Compute the integral $\int_{0}^{-\pi} z \sin(z) \, dz$ along the upper arc of the circle centered at $-\pi/2$ of radius $\pi/2$

a) $2\pi$

b) $-2\pi$

c) $\pi$

d) $-\pi$

e) $0$

IV) Consider the function $f(z) = |z|^2$. Then

a) $f$ is continuous only at 0 but differentiable except at 0

b) $f$ is continuous everywhere and differentiable only at 0

c) $f$ is continuous and differentiable everywhere

d) $f$ is continuous except at 0 and differentiable except at 0

e) $f$ is continuous everywhere and differentiable except at 0

V) Write $f(z)$ for the function $\frac{z}{z-1}$. If $\Omega$ is the region sketched below, compute $\int_{\partial \Omega} f(z) \, dz$

a) $0$

b) $2\pi i$

c) $1$

d) $-1$

e) $-2\pi i$
VI) There are real constants $a, b, c, d$ so that the function: $f(x, y) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$ is holomorphic everywhere. For these constants, the value of the expression $ad - bc$ is

a) 2  
b) 3  
c) 0  
d) 1  
e) 4

VII There are many solutions of the equation $\exp(\frac{1}{2}) = -1$. Write $z_1$ and $z_2$ for the solutions having the biggest absolute value among all solutions of our equation. Then $|z_1| + |z_2|$ is equal to

a) $\pi$  
b) $2\pi$  
c) $1/\pi$  
d) $2/\pi$  
e) $3/\pi$

VIII Consider the equation $z^8 - 2z^4 + 1 = 0$. Then the sum of all its roots equals

a) 1  
b) $-1$  
c) 2  
d) 0  
e) $-2$

IX Consider the equation $\cos z = isinz$. Then

a) There are many solutions, any two differ by an odd multiple of $i\pi$  
b) There are many solutions, any two differ by an odd multiple of $\pi$  
c) There are many solutions, any two differ by an even multiple of $i\pi$  
d) There are many solutions, any two differ by an even multiple of $\pi$  
e) There are no solutions.
X  For the (complicated) curve, $C$, sketched below and traced as sketched, and for the function: $f(z) = \frac{2}{(z-1)(z+1)}$, compute the integral $\int_C f(z) \, dz$.

The answer is

a) $3\pi i$

b) $6\pi i$

c) $2\pi i$

d) $4\pi i$

e) $8\pi i$

XI  For the power series $\sum_{k=0}^{\infty} \frac{k^3z^k}{(3+4i)^k}$, the radius of convergence is

a) 3

b) 5

c) 1

d) 4

e) 2

XII  Consider the power series $\sum_{k=0}^{\infty} k^{2n}z^k$ (here $n$ is an integer independent of $k$), then on the circle of convergence we have

a) When $n > 0$ there are some points of convergence and some of divergence

b) When $n < 0$ there is divergence everywhere

c) When $n = 0$ there are some points of convergence and some of divergence

d) When $n < 0$ there is convergence everywhere

e) You cannot tell, for each $n$ anything can happen.

END OF THE EXAM