Linear Functions

Slope

\[ m = \frac{\text{rise}}{\text{run}} \]

\[ m = \frac{\text{change in } y}{\text{change in } x} \]

As you move from one point on the line to another, to gain a general understanding, we assume we move from left to right.

To move from one point on the line to another, we move from left to right.

\[ \text{rise} \quad \text{run} \]

\[ m > 0 \quad (m \text{ is positive}) \]

\[ m < 0 \quad (m \text{ is negative}) \]

\[ \text{rise} \quad \text{run} \quad m = 0 \quad (\text{horizontal line}) \]

\[ \text{rise} \quad \text{run} \quad m = \text{undefined} \quad (\text{vertical line}) \]

Given 2 points: \((x_1, y_1), (x_2, y_2)\)

Find the slope of the line through the points.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

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\((7, -4), (-7, 3)\)

\[ m = \frac{3 - (-4)}{-7 - 7} \]

\[ m = \frac{7}{-14} \]

\[ m = \frac{1}{2} \]

Parallel Lines have same slope

Perpendicular Lines have opposite sign reciprocal slopes
Equation of a line

Slope Intercept Form
\[ y = mx + b \]

Point - Slope Form
\[ y - y_1 = m(x - x_1) \]

General Form
\[ Ax + By + C = 0 \]
\[ m = -\frac{A}{B} \]

In order to find the equation of the line you need:

a) a point on the line
b) the slope of the line

Find the equation of a line:

a) Given the slope and a point that the lines goes through

6. Find the equation of the line through \((4,7)\), perpendicular to \(2x - 3y = 5\).

a. \( y = \frac{3}{2}x + 6 \)

b. \( y = -\frac{3}{2}x + 13 \)

c. \( y = \frac{2}{3}x + \frac{13}{3} \)

d. \( y = -\frac{3}{2}x + 1 \)

e. \( y = \frac{2}{3}x - \frac{5}{3} \)

\[ 2x - 3y = 5 \]
\[ m = -\frac{A}{B} = -\frac{2}{-3} = \frac{2}{3} \]

\[ y - y_1 = m(x - x_1) \]
\[ y - 7 = \frac{2}{3}(x - 4) \]
\[ y - 7 = -\frac{3}{2}x + 6 \]
\[ y = -\frac{3}{2}x + 13 \]

B
Find the equation of a line:

b) Given 2 points on the line

8. Assume that a student’s grade on an exam is a linear function of the number of hours slept the night before. If student Q slept 7 hours and received a 64 and student K slept 9.5 hours and received an 84, find the linear function relating grade (y), to hours slept (x).

\[ m = \frac{84 - 64}{9.5 - 7} = \frac{20}{2.5} = 8 \]

\[ y - y_1 = m(x - x_1) \]

\[ y - 64 = 8(x - 7) \]

\[ y - 64 = 8x - 56 \]

\[ y = 8x + 8 \]

C

Section 1.4 – Introduction to Functions

Function: A rule that assigns to every input one and only one output.

Domain: The set of all inputs

Range: The set of all outputs
In Exercises 1–4, decide whether the set of figures represents a function from $A$ to $B$.
$A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$
Give reasons for your answers.

1. Each input used once and only once.
   **FUNCTION**

2. Each input used once and only once.
   **FUNCTION**

3. One input (b) not used
   **NOT A FUNCTION**

4. One input (a) used more than once
   **NOT A FUNCTION**

Evaluate a function

$f(x) = -x^2 + 3x + 4$

$f(0) = 4$

$f(1) = -1 + 3 + 4 = 6$

$f(-2) = -4 - 6 + 4 = -6$

$f(t) = -t^2 + 3t + 4$

$f(x+1) = -(x+1)^2 + 3(x+1) + 4$

$= -(x^2 + 2x + 1) + 3x + 3 + 4$

$= -x^2 - 2x - 1 + 3x + 3 + 4$

$= -x^2 + x + 6$
Piecewise Functions

\[ f(x) = \begin{cases} 
\sqrt{x^2 - 4x} & , x < 0 \text{ or } x > 4 \\
-3x & , 0 \leq x \leq 4 \\
x^2 + 1 & \end{cases} \]

Find \( f(-3) \)

\(-3 < 0 \) so use the TOP branch

\[ f(-3) = \sqrt{(-3)^2 - 4(-3)} \]

\[ f(-3) = \sqrt{9 + 12} \]

\[ f(-3) = \sqrt{21} \]

---

Find the domain of a function given just the equation.

Find the \( x \) values that can be used as inputs in a function

Or (in reverse) find the \( x \) values that can NOT be used as inputs in a function and throw those values out

Two DANGERS to be on the lookout for:

1. Dividing by zero.
   
   ✦ Find all values that make the denominator equal to zero and throw those values out.
   
   \( f(x) = \frac{17}{x^2 - 4} \quad x^2 - 4 = 0 \quad x^2 = 4 \quad x = \pm 2 \)

   **Domain**: all real values EXCEPT \( x = \pm 2 \)

2. Taking the square root of a negative number
   
   ✦ Set the expression under the square root symbol \( \geq 0 \).
   
   \( f(x) = \sqrt{2x - 4} \quad 2x - 4 \geq 0 \quad 2x \geq 4 \quad x \geq 2 \)

   **Domain**: \( x \geq 2 \)
Vertical Line Test

Used to find out if you have a function by looking at the graph

Every vertical line can only touch the graph at one (and only one) point.

On the flip side if there is one vertical line that touches the graph more than once, then it is not a function.

<table>
<thead>
<tr>
<th>Vertical Line Test</th>
<th>Vertical Line Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOT A FUNCTION</td>
<td>NOT A FUNCTION</td>
</tr>
</tbody>
</table>

Section 1.6 – Transformations of Functions

\[ f(x) = x^2 \quad f(x) = |x| \quad f(x) = \sqrt{x} \]

\[ f(x) = A(x + B)^2 + C \]
\[ f(x) = A|x + B| + C \]
\[ f(x) = A\sqrt{x + B} + C \]
\[
f(x) = A(x + B)^2 + C
\]

**A:** \(x\)-axis flip and/or shape distortion

- \(A > 1\) or \(A < -1\) ⇒ Stretch in the y direction (thinner)
- \(-1 < A < 1\) ⇒ Shrink in the y direction (wider)
- \(A < 0\) ⇒ \(x\)-axis flip

**B:** Shift in the \(x\) direction
- \(B > 0\) ⇒ Shift left
- \(B < 0\) ⇒ Shift right

**C:** Shift in the \(y\) direction
- \(C > 0\) ⇒ Shift up
- \(C < 0\) ⇒ Shift down

\[
f(x) = -3(x + 4)^2 - 2
\]
$$f(x) = -\frac{1}{2} |x - 3| + 4$$

-1 < A < 1
-1 < A < 1

A < 0
flip

C > 0
Shift Up

B < 0
Shift Right

A > 1
Thinner?
Vertical Stretch

B < 0
Shift Right

C < 0
Shift Down

$$f(x) = 2\sqrt{x - 1} - 3$$

$$f(x) = 2\sqrt{x - 1} - 3$$

$$f(x) = \sqrt{x}$$
Section 1.7 The Algebra of Functions

Let \( f \) and \( g \) be two functions with domains that overlap. For all \( x \) common to both domains,

\[
\begin{align*}
\diamond \quad (f + g)(x) &= f(x) + g(x) \\
\diamond \quad (f - g)(x) &= f(x) - g(x) \\
\diamond \quad (fg)(x) &= f(x) \cdot g(x) \\
\diamond \quad \left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}, \quad g(x) \neq 0 \\
\diamond \quad (f \circ g)(x) &= f(g(x)) \quad \text{(Composition of functions)}
\end{align*}
\]

Section 1.7 The Algebra of Functions

Let \( f(x) = x^4 + \frac{1}{x} \) and \( g(x) = \sqrt{x} \)

Find \( f \left( g(4) \right) \)

Let \( x = 4 \):

\[
g(4) = \sqrt{4} = 2
\]

\[
f(2) = 2^4 + \frac{1}{2} = 16 + \frac{1}{2} = 16.5
\]

Find \( f \left( g(9) \right) \)

Let \( x = 9 \):

\[
g(9) = \sqrt{9} = 3
\]

\[
f(3) = 3^4 + \frac{1}{3} = 81 + \frac{1}{3} = 81 \frac{1}{3}
\]
Section 1.8 Inverse Functions

Symbol

\( f^{-1}(x) \)

The inverse function is the special function that when composed with \( f \) in either direction gives \( x \).

\[ f \left( f^{-1}(x) \right) = x \quad \text{and} \quad f^{-1} \left( f \left( x \right) \right) = x \]

Finding Inverse Functions

1. Replace \( f(x) \) by \( y \)
2. Switch \( x \) and \( y \)
3. Solve the new equation for \( y \)
4. Replace \( y \) by \( f^{-1}(x) \)