We want to be able to find the minima and maxima of functions

**Definition**

A function \( f \) has an **absolute maximum** at \( c \) if \( f(c) \geq f(x) \) for all \( x \) in the domain of \( f \). \( f(c) \) is the **maximum value** of \( f \).

A function \( f \) has an **absolute minimum** at \( c \) if \( f(c) \leq f(x) \) for all \( x \) in the domain of \( f \). \( f(c) \) is the **minimum value** of \( f \).
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A function $f$ has an **absolute minimum** at $c$ if $f(c) \leq f(x)$ for all $x$ in the domain of $f$. $f(c)$ is the **minimum value** of $f$.

**Definition**

A function $f$ has an **local maximum** at $c$ if $f(c) \geq f(x)$ when $x$ is near $c$.

A function $f$ has an **local minimum** at $c$ if $f(c) \leq f(x)$ when $x$ is near $c$. 
**Theorem**

*(Extreme Value Theorem)*

If \( f \) is continuous on a closed interval \([a, b]\), then \( f \) attains an absolute maximum value \( f(c) \) and an absolute minimum value \( f(d) \) at some numbers \( c \) and \( d \) in \([a, b]\).
Theorem

(Fermat’s Theorem)
If \( f \) has a local maximum or minimum at \( c \), and if \( f'(c) \) exists, then \( f'(c) = 0 \).
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Definition

A Critical Number of a function \( f \) is a number \( c \) in the domain of \( f \) such that either \( f'(c) = 0 \) or \( f'(c) \) does not exist.
The Closed Interval Method

To find the absolute maximum and minimum values of a continuous function \( f \) on a closed interval \([a, b]\):

**Step 1:** Find the values of \( f \) at the critical numbers of \( f \) in \((a, b)\).

**Step 2:** Find the values of \( f \) at the endpoints of the interval.

**Step 3:** The largest of the values from step 1 and step 2 is the absolute maximum value; the smallest of the values from step 1 and step 2 is the absolute minimum value.
Linear Approximations

The tangent line at \((a, f(a))\) is an approximation of \(f(x)\) when \(x\) is near \(a\).

The tangent line to \(f(x)\) at the point \((a, f(a))\) is given by the formula

\[
y = f(a) + f'(a)(x - a)
\]

**Definition**

The linearization of \(f\) at \(a\) is given by:

\[
L(x) = f(a) + f'(a)(x - a)
\]