Steps to Solving Optimization Problems

1. Draw a picture representing the problem.
2. Find a formula for the quantity being optimized.
3. Use the information in the problem to express the quantity being optimized in terms of a single variable.
4. Use the first derivative test to find the local minima and maxima.
5. Finish solving the problem.
Example
A farmer has 2400ft of fencing and wants to fence off a rectangular field that boarders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

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Example
A cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

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Example
Find the point on the parabola \( y^2 = 2x \) that is closest to the point \((1, 4)\).

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Example
Find the dimensions of a rectangle of largest area that can be inscribed in an equilateral triangle of side length $L$ if one side of the rectangle lies on the base of the triangle.

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