Math 103 Day 20: The Fundamental Theorem of Calculus and Indefinite Integrals

Ryan Blair

University of Pennsylvania

Thursday November 18, 2010
The Fundamental Theorem of Calculus and Definite Integrals
Properties of Integrals

1. \( \int_{a}^{b} f(x) \, dx = -\int_{b}^{a} f(x) \, dx \)
2. \( \int_{a}^{a} f(x) \, dx = 0 \)
3. \( \int_{a}^{b} c \, dx = c(b - a) \) where \( c \) is any constant.
4. \( \int_{a}^{b}[f(x) + g(x)] \, dx = \int_{a}^{b} f(x) \, dx + \int_{a}^{b} g(x) \, dx \)
5. \( \int_{a}^{b} cf(x) \, dx = c \int_{a}^{b} f(x) \, dx \) where \( c \) is a constant.
6. \( \int_{a}^{b}[f(x) - g(x)] \, dx = \int_{a}^{b} f(x) \, dx - \int_{a}^{b} g(x) \, dx \)
The Fundamental Theorem of Calculus and Definite Integrals

**Theorem**

*(Fundamental Theorem of Calculus, Part 1)* If $f$ is continuous on $[a, b]$, then the function $g$ defined by

$$g(x) = \int_a^x f(t) \, dt \quad a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on $(a, b)$, and $g'(x) = f(x)$. 
The Fundamental Theorem of Calculus and Definite Integrals

**Theorem**

*(Fundamental Theorem of Calculus, Part 1)* If $f$ is continuous on $[a, b]$, then the function $g$ defined by

$$g(x) = \int_a^x f(t) \, dt \quad a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on $(a, b)$, and $g'(x) = f(x)$.

**Theorem**

*(Fundamental Theorem of Calculus, Part 2)* If $f$ is continuous on $[a, b]$, then

$$\int_a^b f(x) \, dx = F(b) - F(a)$$

Where $F$ is any antiderivative of $f$, that is, a function such that $F' = f$. 
Definition

\[ \int f(x) \, dx = F(x) \quad \text{means} \quad F'(x) = f(x) \]
**Exercise** Water flows from the bottom of a storage tank at a rate of 
\[ r(t) = 200 - 4t \] liters per minute, where \( 0 \leq t \leq 50 \). Find the amount of water that flows from the tank during the first 10 minutes.