Math 103 Day 6: Derivative Rules

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1. Derivative Rules
**Formula 1:** When $c$ is a constant

$$
\frac{d}{dx}(c) = 0
$$
Formula 2:

\[
\frac{d}{dx}(x) = 1
\]
Formula 3: When $n$ is a positive integer,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$
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**fact:**

$$(x - a)^n = (x - a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + ... + a^{n-2}x + a^{n-1})$$
**Formula 4:** (General Power Rule) When $n$ is any real number,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$
**Formula 5:** If $c$ is a constant and $f$ is differentiable, then

$$
\frac{d}{dx}(cf(x)) = c \frac{d}{dx}(f(x))
$$
**Formula 6:** (Sum Rule) If $g$ and $f$ are differentiable, then

\[
\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]
\]
**Formula 7:** (Difference Rule) If $g$ and $f$ are differentiable, then

$$
\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)]
$$
Formula 8: (Product Rule) If $f$ and $g$ are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$
**Formula 9:** (Quotient Rule) If $f$ and $g$ are differentiable, then

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} (f(x)) - f(x) \frac{d}{dx} (g(x))}{(g(x))^2}$$
Theorem

If $f(x) = \sin(x)$, then $f'(x) = \cos(x)$. 
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This is challenging to prove, so we need some lemmas.
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Lemma

\[
\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1
\]

Lemma

\[
\lim_{\theta \to 0} \frac{(\cos(\theta) - 1)}{\theta} = 0
\]