Outline

1 Review

2 One-Sided Limits
Definition of Limit

**Definition**

If \( f(x) \) is arbitrarily close to \( L \) for all \( x \) sufficiently close to \( x_0 \), we say \( f \) approaches the **limit** \( L \) as \( x \) approaches \( x_0 \) and write:

\[
\lim_{x \to x_0} f(x) = L
\]
Definition of Limit

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Last time we saw

1. Limit laws
2. Theorems regarding polynomials and rational functions
3. How to evaluate a limit if there is a zero in the denominator
The Sandwich Theorem

**Theorem**

If \( f(x) \leq g(x) \leq h(x) \) when \( x \) is near \( c \) and

\[
\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L
\]

then \( \lim_{x \to c} g(x) = L \)
The Sandwich Theorem

Theorem

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then \( \lim_{x \to c} g(x) = L \)

Evaluate:

\[
\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)
\]
Definition of One-Sided Limit

Definition

If \( f(x) \) is arbitrarily close to \( L \) for all \( x \) sufficiently close to \( c \) and greater than \( c \), we say \( f \) approaches the right-hand limit \( L \) as \( x \) approaches \( c \) and write:

\[
\lim_{x \to c^+} f(x) = L
\]
One-Sided Limits

Definition of One-Sided Limit

Definition

If \( f(x) \) is arbitrarily close to \( L \) for all \( x \) sufficiently close to \( c \) and greater than \( c \), we say \( f \) approaches the \textbf{right-hand limit} \( L \) as \( x \) approaches \( c \) and write:

\[
\lim_{x \to c^+} f(x) = L
\]

Definition

If \( f(x) \) is arbitrarily close to \( L \) for all \( x \) sufficiently close to \( c \) and less than \( c \), we say \( f \) approaches the \textbf{left-hand limit} \( L \) as \( x \) approaches \( c \) and write:

\[
\lim_{x \to c^-} f(x) = L
\]
Theorem

\[ \lim_{x \to c} f(x) = L \]

if and only if

\[ \lim_{x \to c^+} f(x) = L \quad \text{and} \quad \lim_{x \to c^-} f(x) = L. \]
Theorem

\[ \lim_{x \to 0} \frac{\sin(x)}{x} = 1 \]