Math 103: Derivatives and Derivative Rules

Ryan Blair

University of Pennsylvania

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Outline

1. Review
2. Derivatives as Functions
3. Derivative Rules
Limits Involving Infinity

1. Tangent lines to functions.
2. Secant lines to functions.
3. Finding the slopes of tangent lines.
4. Derivatives of functions.
Interpretations of Derivative at a Point

\[ \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \]
Interpretations of Derivative at a Point

\[ \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \]

1. The slope of the graph \( y = f(x) \) at \( x = a \).
2. The slope of the tangent line to the curve \( y = f(x) \) at \( x = a \).
3. The rate of change of \( f(x) \) with respect to \( x \) at \( x = a \).
4. The derivative of \( f(x) \) at \( x = a \).
Derivative as a function

Definition

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]
Derivative as a function

**Definition**

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

**Alternative Form.**

\[ f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} \]
Theorem

*If* $f$ *is differentiable at* $a$, *then* $f$ *is continuous at* $a$. 
Theorem

If $f$ is differentiable at $a$, then $f$ is continuous at $a$.

To show $f$ is continuous at $a$, we must show

$$\lim_{x \to a} f(x) = f(a).$$
Theorem

If \( f \) is differentiable at \( a \), then \( f \) is continuous at \( a \).

To show \( f \) is continuous at \( a \), we must show

\[
\lim_{x \to a} f(x) = f(a).
\]

However, using our limit laws, this is equivalent to showing

\[
\lim_{x \to a} (f(x) - f(a)) = 0.
\]
Theorem

If $f$ is differentiable at $a$, then $f$ is continuous at $a$.

To prove the theorem we will assume

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

and we will show

$$\lim_{x \to a}(f(x) - f(a)) = 0.$$
**Formula 1:** When $c$ is a constant

$$\frac{d}{dx}(c) = 0$$
**Formula 2:** When $n$ is a positive integer,

$$
\frac{d}{dx}(x^n) = nx^{n-1}
$$
Formula 2: When $n$ is a positive integer,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

fact: $x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \ldots + a^{n-2}x + a^{n-1})$
**Formula 3:** (General Power Rule) When $n$ is any real number,

\[
\frac{d}{dx}(x^n) = nx^{n-1}
\]
**Formula 4:** If $c$ is a constant and $f$ is differentiable, then

\[
\frac{d}{dx}(cf(x)) = c \frac{d}{dx}(f(x))
\]
**Formula 5:** (Sum Rule) If $g$ and $f$ are differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$
Formula 6: (Exponential Functions)

\[
\frac{d}{dx}[a^x] = \ln(a)a^x
\]
Formula 7: (Product Rule) If $f$ and $g$ are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$
**Formula 8:** (Quotient Rule) If $f$ and $g$ are differentiable, then

$$
\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx}(f(x)) - f(x) \frac{d}{dx}(g(x))}{(g(x))^2}
$$