Outline

1. Spanning

2. Linear Independence
Today’s Goals

1. Be able to determine if a set of vectors spans a vector subspace.
2. Be able to determine if a set of vectors is linearly independent.
The span of a set of vectors

Definition

A set of vectors $v_1, v_2, ..., v_n$ spans a vector space $V$ if every vector in $V$ can be written as $c_1v_1 + c_2v_2 + ... + c_nv_n$ where $c_i$ is a scalar for $1 \leq i \leq n$.

In this case we say $V$ is spanned by $v_1, v_2, ..., v_n$. 
The span of a set of vectors

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Exercise: Show that $\{(1, 2, 1), (1, 0, 1), (0, 1, 1)\}$ spans $\mathbb{R}^3$. 
The span of a set of vectors

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In this case we say $V$ is spanned by $v_1, v_2, \ldots, v_n$.

**Exercise:** Show that $\{(1, 2, 1), (1, 0, 1), (0, 1, 1)\}$ spans $\mathbb{R}^3$.

**Exercise:** Show that $\{(1, 2, 1), (1, 0, 1), (0, 1, 0)\}$ does not span $\mathbb{R}^3$. 
Linear Independence

Definition
Let $v_1, \ldots, v_m$ be vectors in a vector space $V$. The set $S = \{v_1, \ldots, v_m\}$ is **linearly independent** if $c_1 v_1 + c_2 v_2 + \ldots + c_n v_n = 0$ implies $c_1 = c_2 = \ldots = c_n = 0$.

If there exists a non trivial solution to $c_1 v_1 + c_2 v_2 + \ldots + c_n v_n = 0$ we say the set $S$ is linearly dependant.
Linear Independence

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If there exists a non trivial solution to $c_1 v_1 + c_2 v_2 + ... + c_n v_n = 0$ we say the set $S$ is linearly dependant.

**Exercise:** Are the following vectors linearly independent?

$$<1, 2, 1>, <1, 0, 1>, <0, 1, 1>$$