Math 240: The Rank Nullity Theorem

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The Rank-Nullity Theorem
Today’s Goals

1. Understand the proof of the Rank-Nullity Theorem.
2. Be able to apply the Rank-Nullity theorem.
The Rank-Nullity Theorem

Definition

If $A$ is an $m \times n$ matrix, then the **null space** of $A$ is the vector space of solutions to $Ax = 0$. In other words,

$$nullspace(A) = \{ x \in \mathbb{R}^n : Ax = 0 \}$$

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The dimension of $nullspace(A)$ is the **nullity** of $A$ and is denoted by $nullity(A)$. 

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Theorem

(The Rank-Nullity Theorem) For any $m \times n$ matrix $A$,

$$rank(A) + nullity(A) = n$$