Outline

1 Operations on Matrices
Goals

1. Matrix basics
2. Add and subtract matrices
3. Multiply a matrix by a scalar
4. Multiply matrices
5. Take the transpose of a matrix
6. Special types of matrices
7. Matrix properties
Definition

A **matrix** is a rectangular array of numbers or functions with $m$ rows and $n$ columns.
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\[
X = \begin{pmatrix}
  x_{1,1} & x_{1,2} & \ldots & x_{1,n} \\
  x_{2,1} & x_{2,2} & \ldots & x_{2,n} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{m,1} & x_{m,2} & \ldots & x_{m,n}
\end{pmatrix} = (x_{i,j})_{m \times n}
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The **dimension** of a matrix is (the number of Rows) $\times$ (the number of columns).
A Quick Review

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\end{pmatrix} = (x_{i,j})_{m \times n}
\]

The **dimension** of a matrix is (the number of Rows) × (the number of columns). Two Matrices are equal if they have the same dimension and corresponding entries are equal.
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4. Matrix Transpose: \((a_{ij})^T_{m\times n} = (a_{ji})_{n\times m}\) (Rows of \(A\) become columns of \(A^T\) and columns of \(A\) become rows of \(A^T\).)
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**Definition**

The **identity matrix of dimension** $n$, denoted $I_n$, is the $n \times n$ diagonal matrix where all the diagonal entries are 1.
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Definition
The **trace** of a square matrix is the sum of the diagonal entries.
Matrix Properties

Let $A$ and $B$ be $m \times n$ matrices. Let $k$ and $p$ be scalars.

1. $A + B = B + A$
2. $A + (B + C) = (A + B) + C$
3. $k(A + B) = kA + kB$
4. $(k + p)A = kA + pA$

Let $0$ be the $m \times n$ matrix of all zeros

1. $A + 0 = A$
2. $A - A = 0$
3. $kA = 0$ implies $k = 0$ or $A = 0$. 
More Matrix Properties

1. \( A(BC) = (AB)C \)
2. \( A(B + C) = AB + AC \)
3. \( (A + B)C = AC + BC \)
4. \( k(AB) = (kA)B = A(kB) \)
5. \( I_mA = A \)
6. \( AI_n = A \)
Even More Matrix Properties

1. $(A^T)^T = A$
2. $(kA)^T = kA^T$
3. $(A + B)^T = A^T + B^T$
4. $(AB)^T = B^T A^T$