Outline

1. Review

2. Today’s Goals
Stokes’ Theorem

Theorem

Let $S$ be an nice oriented surface bounded by a nice curve $C$. Let $F = Pi + Qj + Rk$ be a nice vector field. If $C$ is traversed in the positive direction and $T$ is the unit tangent vector to $C$ then

$$\oint_C F \circ dr = \oint_C (F \circ T) ds = \int \int_S (\text{curl}(F) \circ n) dS$$

where $n$ is the unit normal to $S$ in the direction of the orientation of $S$.

Review Question: Let $F = \langle y^2, 2z + x, 2y^2 \rangle$. Find a plane $ax + by + cz = 0$ such that $\oint_C F \circ dr = 0$ for every smooth simple closed curve $C$ in the plane.
1. Understand how to use the Divergence Theorem.
Today's Goals

**Divergence Theorem**

Let $D$ be a **nice** region in 3-space with **nice** boundary $S$ oriented outward. Let $F$ be a **nice** vector field. Then

$$
\int \int_S (F \circ n) dS = \int \int \int_D \text{div}(F) dV
$$

where $n$ is the unit normal vector to $S$. 

**Theorem**
Divergence Theorem

**Theorem**

Let $D$ be a **nice** region in 3-space with **nice** boundary $S$ oriented outward. Let $F$ be a **nice** vector field. Then

$$\iiint_S (F \circ n) dS = \iiint_D \text{div}(F) dV$$

where $n$ is the unit normal vector to $S$.

**Example** Find the flux of $F = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$ outward through the surface of the cube cut from the first octant by the planes $x = 1$, $y = 1$ and $z = 1$. 
Divergence Theorem

**Theorem**

Let $D$ be a **nice** region in 3-space with **nice** boundary $S$ oriented outward. Let $F$ be a **nice** vector field. Then

$$\int \int S (F \circ n) dS = \int \int \int_D \text{div}(F) dV$$

where $n$ is the unit normal vector to $S$.

**Example:** Use the divergence theorem to evaluate $\int \int S (F \cdot n) dS$ where $F = \langle x + y, z, z - x \rangle$ and $S$ is the boundary of the region between $z = 9 - x^2 - y^2$ and the $xy$-plane.
Divergence Theorem

Let $D$ be a closed and bounded region in 3-space with a piecewise smooth boundary $S$ that is oriented outward. Let $F(x, y, z) = P(x, y, z)i + Q(x, y, z)j + R(x, y, z)k$ be a vector field for which $P$, $Q$ and $R$ are continuous and have continuous first partial derivatives in a region of 3-space containing $D$. Then

$$\int \int_S (F \circ n) dS = \int \int \int_D \text{div}(F) dV$$

where $n$ is the unit normal vector to $S$. 

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Divergence Theorem

**Theorem**  
Let $D$ be a closed and bounded region in 3-space with a piecewise smooth boundary $S$ that is oriented outward. Let $F(x, y, z) = P(x, y, z)i + Q(x, y, z)j + R(x, y, z)k$ be a vector field for which $P$, $Q$ and $R$ are continuous and have continuous first partial derivatives in a region of 3-space containing $D$. Then

$$
\int \int_S (F \circ n) dS = \int \int \int_D \text{div}(F) dV
$$

where $n$ is the unit normal vector to $S$.

**Example** Find the outward flux of

$$
\frac{\langle x, y, z \rangle}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}
$$

across the region $D$ given by $0 \leq a^2 \leq x^2 + y^2 + z^2 \leq b^2$. 

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