Outline

1. Review

2. Today's Goals
Review of Last Time

1. Found power series solutions to D.E.s at ordinary points.
Solving D.E.s Using Power Series

Given the differential equation \( y'' + P(x)y' + Q(x)y = 0 \), substitute
\[
y = \sum_{n=0}^{\infty} c_n(x - a)^n
\]
and solve for the \( c_n \) to find a power series solution centered at \( a \).
Given the differential equation \( y'' + P(x)y' + Q(x)y = 0 \), substitute

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and solve for the \( c_n \) to find a power series solution centered at \( a \).

Solve the following D.E.

\[
y'' - 2xy' + y = 0
\]
Today's Goals

1. Find power series solutions to D.E.s at singular points.
Given a differential equation \( y'' + P(x)y' + Q(x)y = 0 \)

**Definition**

A point \( x_0 \) is an **ordinary point** if both \( P(x) \) and \( Q(x) \) are analytic at \( x_0 \). If a point is not ordinary it is a **singular point**.
Given a differential equation $y'' + P(x)y' + Q(x)y = 0$

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A point $x_0$ is an **ordinary point** if both $P(x)$ and $Q(x)$ are analytic at $x_0$. If a point is not ordinary it is a **singular point**.

**Definition**

A point $x_0$ is a **regular singular point** if the functions $(x - x_0)P(x)$ and $(x - x_0)^2 Q(x)$ are both analytic at $x_0$. Otherwise $x_0$ is irregular.
Theorem

(Frobenius’ Theorem)

If \( x_0 \) is a regular singular point of \( y'' + P(x)y' + Q(x)y = 0 \), then there exists a solution of the form

\[
y = \sum_{n=0}^{\infty} c_n (x - x_0)^{n+r}
\]

where \( r \) is some constant to be determined and the power series converges on a non-empty open interval containing \( x_0 \).
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where \( r \) is some constant to be determined and the power series converges on a non-empty open interval containing \( x_0 \).

To solve \( y'' + P(x)y' + Q(x)y = 0 \) at a regular singular point \( x_0 \), substitute

\[
y = \sum_{n=0}^{\infty} c_n (x - x_0)^{n+r}
\]

and solve for \( r \) and the \( c_n \) to find a series solution centered at \( x_0 \).