The examination consists of 10 problems each worth 10 points; answer all of them. The first eight are multiple choice, NO PARTIAL CREDIT given, but NO PENALTY FOR GUESSING. To answer, CIRCLE THE ENTIRE STATEMENT YOU DEEM CORRECT in the problem concerned. No work is required to be shown on the multiple choice problems, use your bluebooks for computations and scratch work. The last two problems are long answer; work must be shown for credit and PARTIAL CREDIT is given. USE THE PAGES IN YOUR EXAM TO SHOW YOUR WORK ON THESE TWO PROBLEMS. DO NOT HAND IN YOUR BLUE BOOKS. No books, tables, notes, calculators, computers, phones or electronic equipment allowed; one 3 inch by 5 inch index card handwritten both sides allowed, no substitutions of this aid allowed.

YOUR NAME (print please):

YOUR PENN ID NUMBER:

YOUR SIGNATURE:

YOUR RECITATION SECTION DAY/TIME:  DAY:  TIME:

SCORE:

MULTIPLE CHOICE PROBLEMS
I) Let \( \gamma \) be the straight line going from \( i \) to \( 1 + (\pi/2)i \). Compute the line integral \( \int_{\gamma} \cos(4z) \, dz \).

a) \( (1/4)[\cos^2(2\pi i) + \sin^2(4) - \sin(4i)] \)

b) \( (1/4)[\sin(4)\cos(2\pi i) + \cos(4)\sin(2\pi i) - \sin(4i)] \)

c) \( (1/4)[\sin^2(2\pi i) + \cos^2(4) - \sin(4i)] \)

d) \( (1/4)[\cos^2(4 + 2\pi i) - \sin(4i)] \)

e) \( (1/4)[\sin^2(4 + 2\pi i) - \sin(4i)] \)
II) When we compute \((-1 + i)^{\frac{1}{4}}\), we find

a) The absolute value of all the solutions is \(2^{\frac{1}{4}}\)

b) The arguments of the solutions are \(3\pi/4, 5\pi/4, 7\pi/4, 9\pi/4\)

c) The absolute value of all the solutions is \(\sqrt{2}\)

d) The arguments of the solutions are \(3\pi/16, 11\pi/16, 19\pi/16, 27\pi/16\)

e) The arguments of the solutions are \(0, \pi, 3\pi/2, 5\pi/2\).

III) For the value or values of the complex number(s) \((1 + i)^i\), we find

a) All these lie on a circle of radius \(\frac{\log(2)}{2}\)

b) All these lie on a line of slope 1

c) All these lie on a line of slope \(\tan\left(\frac{\log(2)}{2}\right)\)

d) All these values are real numbers

e) All these values are pure imaginary numbers.

IV) Describe the collection of all complex numbers that satisfy the inequality \(|\text{Re}(z)| + |\text{Im}(z)| \leq |z|\).

a) All complex numbers

b) The complex numbers on the unit circle

c) The complex numbers in the right half plane \((\text{Re}(z) > 0)\)

d) The complex numbers in the upper half plane \((\text{Im}(z) > 0)\)

e) The complex numbers on both the real and imaginary axes.
V) Consider the complex function

\[ f(z) = \frac{(z-3-4i)}{z^2-6z+25}. \]

At what point or points is \( f(z) \) NOT holomorphic?

a) No points, \( f(z) \) is holomorphic everywhere

b) Two points: \( z = 3 + 4i, \ z = 3 - 4i \)

c) Only at \( z = 3 - 4i \)

d) Only at \( z = 3 + 4i \)

e) Only on the real axis.

VI) Find all points, \( z \), in the complex plane for which \( \cos(z) = \sin(z) \):

a) Only for complex numbers, \( z \), where \( \text{Re}(z) = \text{Im}(z) \)

b) Only on the real axis for points, \( x \), where \( \cos(x) = \sin(x) \)

c) At the points in b) and at the corresponding points, \( iy \), on the imaginary axis where \( \cos(y) = \sin(y) \)

d) On the unit circle

e) Only on the imaginary axis for points, \( iy \), where \( \cos(y) = \sin(y) \).

VII) If \( \gamma \) is the unit circle, compute \( \int_\gamma (z^3 + z^2 + \text{Im}(z)) \, dz \)

a) \( 2\pi \)

b) \( 2\pi i \)

c) \( -\pi i \)

d) \( -\pi \)

e) 0
VIII) Let $\gamma$ denote the top half of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, traced from $(-3, 0)$ to $(3, 0)$. Compute the integral $\int_\gamma \cos^2(\pi z) \, dz$.

a) $3\pi$

b) $3$

c) $-\pi$

d) $2\pi$

e) 2

LONG ANSWER PROBLEMS (show work below and, if necessary, on the back of pages)

IX) Write $\gamma$ for the circle given by $|z| = 2$ traced counter clockwise. Compute the integral $\int_\gamma \frac{\bar{z}}{(z-1)(z-3)} \, dz$. 
X) Let $\gamma$ be the boundary of the region $|z - 1| < \frac{1}{2}$. Compute the integral $\int_{\gamma} \frac{\cos(\pi z)}{z^3 - z^2} \, dz$. 

END OF THE EXAM