Math 361- Quiz 6

March 25, 2014

1. (Worth 10) Consider the parameter space $D = \{(\theta, z) : 0 \leq z \leq 1, 0 \leq \theta \leq \pi/2\}$ and the two-surface $\Phi$

$$\Phi : D \to \mathbb{R}^3 \quad \Phi(\theta, z) = (2 \cos(\theta), 2 \sin(\theta), z) = (x, y, z).$$

Consider the two form $\omega = (z + y)dx \wedge dy + xdz \wedge dx$. Briefly describe the surface $\Phi(D)$ and compute $\int_{\Phi} \omega$.

**Proof.** First, we describe the region $\Phi(D)$, which is easier by just drawing it. The 2-surface is the circular side of the quarter cylinder of radius 2 and height 1 in the list octant. We then compute the following integral:

$$\int_{\Phi} \omega = \int_D 0d\theta dz + \int_D (2 \cos(\theta))(2 \sin(\theta))d\theta dz = 2 \int_0^1 \int_0^{\pi/2} \sin(2\theta)d\theta dz = 2$$

$\square$