Inducing Approximately Optimal Flow using Truthful Mediators

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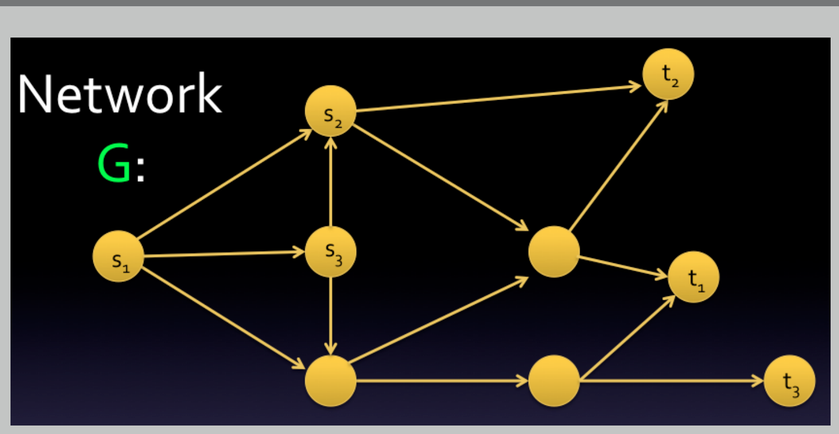
Routing Games



- Losses on each edge $\ell_e(y_e)$.
- Player i routes one unit of flow from s_i to t_i.
- Want selfish players to route optimally.

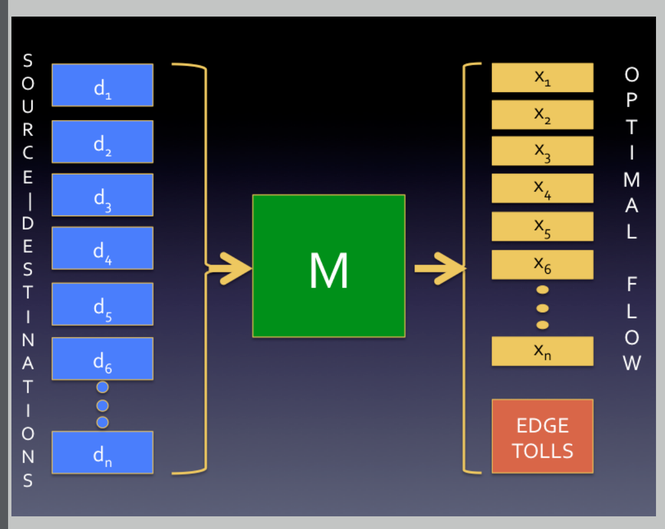
Classical Approach - Impose Tolls

- A Mediator **M** may enforce tolls on each edge so selfish players route optimally.
- New edge losses: $\ell'_e(y_e) = \ell_e(y_e) + \tau_e$.

Enter Mechanism Design

- The tolls the mechanism computes depends on the players' demands.
- Want players to truthfully report their demands so mediator can compute the correct tolls.

Introduce a Mediator that can Enforce Tolls



Weakly Mediated Game

Players:

- may bypass **M**.
- may misreport to **M**.
- may not follow **M**'s suggested route.
- must pay edge tolls.

Main Result

We develop a mediator such that for Large Games:

- Reporting truthfully and following the suggested action of **M**, i.e. **good behavior**, is an (asymptotic) ex - post Nash equilibrium and
- The resulting flow has cost $\mathbf{(1 + o(1))OPT}$.

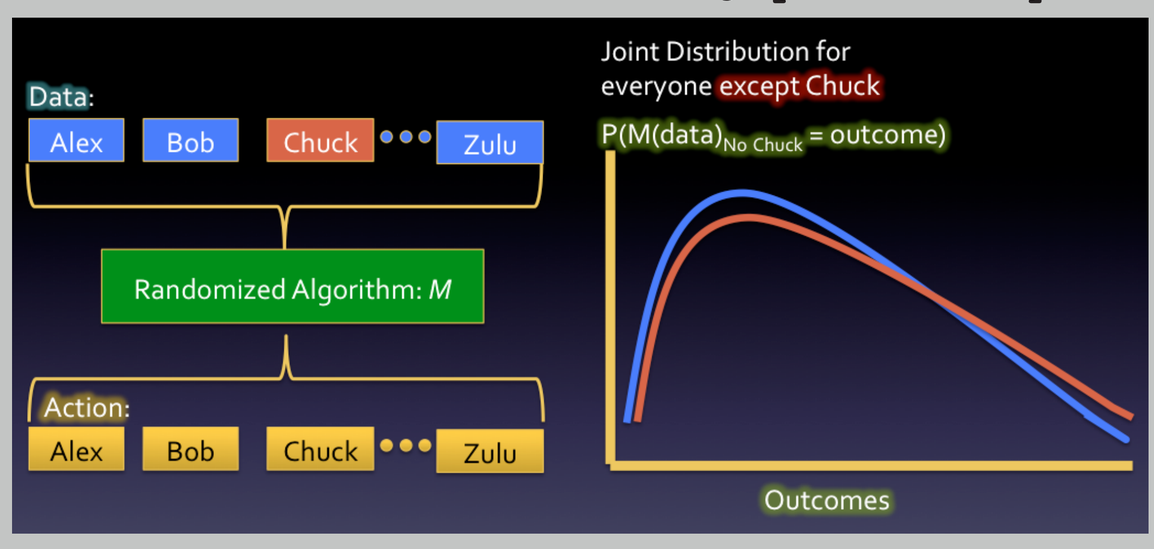
Main Assumption - Large Games

Large Game

Any player has a small $\mathbf{o(1)}$ impact on the costs of others as $\mathbf{n} \to \infty$.

Useful Tool - Joint Differential Privacy

Joint Differential Privacy [KPRU'14]

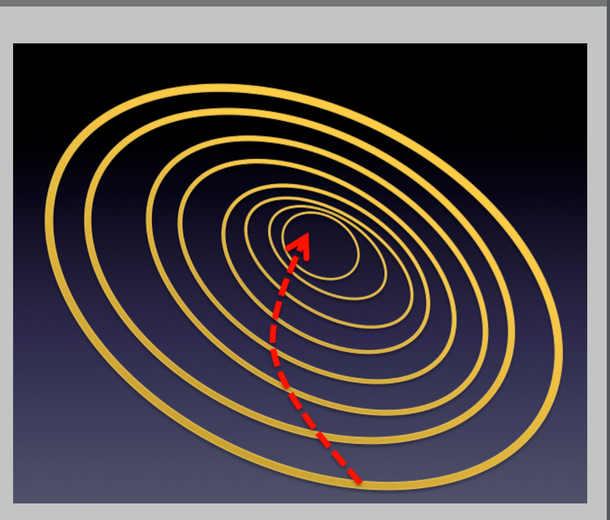


- Controls the impact a single player has on the outcome distribution for the other players.
- No real "privacy" concerns here, but still useful!

Novel Technique - Private Gradient Descent

We need to solve the convex program in a way that is joint differentially private in the data **s**.

$$\begin{array}{ll} \textbf{min} & \text{Total Cost of } \mathbf{x} \\ \text{s.t.} & \mathbf{x} \in \underbrace{\mathcal{F}(\mathbf{s})}_{\text{feasible flow}} \end{array}$$



Conclusion and Open Problem

- We design a weak mediator **M** such that it is an asymptotic ex-post Nash equilibrium for players to truthfully report demands to **M** and follow its suggestion, which results in a nearly optimal flow.
- **Open Problem**: For **any** large game of incomplete information, can we construct a weak mediator such that:
  - **good behavior** is an ex-post NE and
  - players play a NE of the complete information game by following the mediator's suggestion?