**Inducing Approximately Optimal Flow using Truthful Mediators**

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**NYCE 2014**

### Routing Games

- Losses on each edge $\ell_{e}(y_{e})$.
- Player $i$ routes one unit of flow from $s_{i}$ to $t_{i}$.
- Want selfish players to route optimally.

### Classical Approach - Impose Tolls

- A Mediator $M$ may enforce tolls on each edge so selfish players route optimally.
- New edge losses: $\ell'_{e}(y_{e}) = \ell_{e}(y_{e}) + \tau_{e}$.

### Enter Mechanism Design

- The tolls the mechanism computes depends on the players’ demands.
- Want players to truthfully report their demands so mediator can compute the correct tolls.

### Introduce a Mediator that can Enforce Tolls

#### Weakly Mediated Game

**Players:**
- may bypass $M$.
- may misreport to $M$.
- may not follow $M$’s suggested route.
- must pay edge tolls.

#### Main Result

We develop a mediator such that for Large Games:
- Reporting truthfully and following the suggested action of $M$, i.e. good behavior, is an (asymptotic) ex-post Nash equilibrium and
- The resulting flow has cost $(1 + o(1))\text{OPT}$.

### Main Assumption - Large Games

**Large Game**

Any player has a small $o(1)$ impact on the costs of others as $n \to \infty$.

### Useful Tool - Joint Differential Privacy

**Joint Differential Privacy [KPRU’14]**

- Controls the impact a single player has on the outcome distribution for the other players.
- No real “privacy” concerns here, but still useful!

### Novel Technique - Private Gradient Descent

We need to solve the convex program in a way that is joint differentially private in the data $s$.

\[
\min \quad \text{Total Cost of } x \\
\text{s.t.} \quad x \in \mathcal{F}(s) \\
\quad \text{feasible flow}
\]

### Conclusion and Open Problem

- We design a weak mediator $M$ such that it is an asymptotic ex-post Nash equilibrium for players to truthfully report demands to $M$ and follow its suggestion, which results in a nearly optimal flow.
- **Open Problem:** For any large game of incomplete information, can we construct a weak mediator such that:
  - good behavior is an ex-post NE and
  - players play a NE of the complete information game by following the mediator’s suggestion?