BIBLIOGRAPHY OF ROBERT M. STRAIN

[**48**] Eduardo García-Juárez, Yoichiro Mori, and Robert M. Strain, *The Peskin Problem with Viscosity Contrast*, **preprint** (2020), 54 pages, http://arxiv.org/abs/2009.03360 ☠.

Abstract: The Peskin problem models the dynamics of a closed elastic filament immersed in an incompressible fluid. In this paper, we consider the case when the inner and outer viscosities are possibly different. This viscosity contrast adds further non-local effects to the system through the implicit non-local relation between the net force and the free interface. We prove the first global well-posedness result for the Peskin problem in this setting. The result applies for medium size initial interfaces in critical spaces and shows instant analytic smoothing. We carefully calculate the medium size constraint on the initial data. These results are new even without viscosity contrast.

[**47**] Francisco Gancedo, Eduardo García-Juárez, Neel Patel, and Robert M. Strain, *Global regularity for gravity unstable Muskat bubbles*, **preprint** (2020), 81 pages, http://arxiv.org/abs/1902.02318 ☠.

Abstract: In this paper, we study the dynamics of fluids in porous media governed by Darcy's law: the Muskat problem. We consider the setting of two immiscible fluids of different densities and viscosities under the influence of gravity in which one fluid is completely surrounded by the other. This setting is gravity unstable because along a portion of the interface, the denser fluid must be above the other. Surprisingly, even without capillarity, the circle-shaped bubble is a steady state solution moving with vertical constant velocity determined by the density jump between the fluids. Taking advantage of our discovery of this steady state, we are able to prove global in time existence and uniqueness of dynamic bubbles of nearly circular shapes under the influence of surface tension. We prove this global existence result for low regularity initial data. Moreover, we prove that these solutions are instantly analytic and decay exponentially fast in time to the circle.

[**46**] James Chapman, Jin Woo Jang, and Robert M. Strain, *On the Determinant Problem for the Relativistic Boltzmann Equation*, **preprint** (2020), 27 pages, http://arxiv.org/abs/2006.02540 ☠.

Abstract: This article considers a long-outstanding open question regarding the Jacobian determinant for the relativistic Boltzmann equation in the center-of-momentum coordinates. For the Newtonian Boltzmann equation, the center-of-momentum coordinates have played a large role in the study of the Newtonian non-cutoff Boltzmann equation, in particular we mention the widely used cancellation lemma by Alexandre et al. (Arch. Ration. Mech. Anal. 152(4):327–355, 2000). In this article we calculate

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specifically the very complicated Jacobian determinant, in ten variables, for the relativistic collision map from the momentum p to the post collisional momentum p'; specifically we calculate the determinant for $p \mapsto u = \theta p' + (1-\theta)p$ for $\theta \in [0,1]$. Afterwards we give an upper-bound for this determinant that has no singularity in both p and q variables. Next we give an example where we prove that the Jacobian goes to zero in a specific pointwise limit. We further explain the results of our numerical study which shows that the Jacobian determinant has a very large number of distinct points at which it is machine zero. This generalizes the work of Glassey-Strauss (Transport Theory Statist. Phys. 20(1):55–68, 1991) and Guo-Strain (Comm. Math. Phys. 310(3):649–673, 2012). These conclusions make it difficult to envision a direct relativistic analog of the Newtonian cancellation lemma in the center-of-momentum coordinates.

[**45**] Renjun Duan, Shuangqian Liu, Shota Sakamoto, and Robert M. Strain, *Global mild solutions of the Landau and non-cutoff Boltzmann equations*, Comm. Pure Appl. Math. **published online** (2020), 89 pages, http://arxiv.org/abs/1904.12086, http://dx.doi.org/10.1002/cpa.21920.

Abstract: This paper proves the existence of small-amplitude global-in-time unique mild solutions to both the Landau equation including the Coulomb potential and the Boltzmann equation without angular cutoff. Since the well-known works (Guo, 2002) and (Gressman-Strain-2011, AMUXY-2012) on the construction of classical solutions in smooth Sobolev spaces which in particular are regular in the spatial variables, it still remains an open problem to obtain global solutions in an $L^\infty_{x,v}$ framework, similar to that in (Guo-2010), for the Boltzmann equation with cutoff in general bounded domains. One main difficulty arises from the interaction between the transport operator and the velocity-diffusion-type collision operator in the non-cutoff Boltzmann and Landau equations; another major difficulty is the potential formation of singularities for solutions to the boundary value problem. In the present work we introduce a new function space with low regularity in the spatial variable to treat the problem in cases when the spatial domain is either a torus, or a finite channel with boundary. For the latter case, either the inflow boundary condition or the specular reflection boundary condition is considered. An important property of the function space is that the $L^\infty_T L^2_v$ norm, in velocity and time, of the distribution function is in the Wiener algebra $A(\Omega)$ in the spatial variables. Besides the construction of global solutions in these function spaces, we additionally study the large-time behavior of solutions for both hard and soft potentials, and we further justify the property of propagation of regularity of solutions in the spatial variables.

[**44**] Renjun Duan, Shuangqian Liu, Shota Sakamoto, and Robert M. Strain, *Global solutions to the Boltzmann equation without angular cutoff and the Landau equation with Colomb potential*, RIMS Kôkyûroku Bessatsu **B82** (2020), 29–46, proceedings of symposium on "Regularity and Asymptotic Analysis for Critical Cases of Partial Differential Equations", http://www.kurims.kyoto-u.ac.jp/~kenkyubu/bessatsu.html.

Abstract: This report succinctly summarizes results proved in the authors' recent work (2019) where the unique existence of solutions to the Boltzmann equation without angular cut-off and the Landau equation with Coulomb potential are studied in a perturbation framework. A major feature is the use of the Wiener space $A(\Omega)$, which can be expected to play a similar role to L^∞. Compared to the L^2-based solution spaces that were employed for prior known results, this function space enables us to establish a new global existence theory. One further feature is that, not only an initial value problem, but also an initial boundary value problem whose boundary conditions can be regarded as physical boundaries in some simple situation, are considered for both equations. In addition to unique existence, large-time behavior of the solutions and

propagation of spatial regularity are also proved. In the end of report, key ideas of the proof will be explained in a concise way.

[**43**] Jin Woo Jang, Robert M. Strain, and Seok-Bae Yun, *Propagation of uniform upper bounds for the spatially homogeneous relativistic Boltzmann equation*, **preprint** (2019), 31 pages, http://arxiv.org/abs/1907.05784 .

Abstract: In this paper, we prove the propagation of uniform upper bounds for the spatially homogeneous relativistic Boltzmann equation. These L^∞ bounds have been known to be a challenging open problem in relativistic kinetic theory. To accomplish this, we establish two types of estimates for the gain part of the collision operator: first, we prove a potential type estimate and a relativistic hyper-surface integral estimate. We then combine those estimates using the relativistic counter-part of the Carleman representation to derive uniform control of the gain term for the relativistic collision operator. This allows us to prove the desired propagation of the uniform bounds of the solution. We further present two applications of the propagation of the uniform upper bounds: first we give another proof of the Boltzmann H-theorem, and second we prove the asymptotic convergence of solutions to the relativistic Maxwellian equilibrium.

[**42**] Francisco Gancedo, Eduardo García-Juárez, Neel Patel, and Robert M. Strain, *On the Muskat problem with viscosity jump: Global in time results*, Adv. Math. **345** (2019), 552–597, http://arxiv.org/abs/1710.11604 , http://dx.doi.org/10.1016/j.aim.2019.01.017 , Zbl 07021548, MR 3899970.

Abstract: The Muskat problem models the filtration of two incompressible immiscible fluids of different characteristics in porous media. In this paper, we consider both the 2D and 3D setting of two fluids of different constant densities and different constant viscosities. In this situation, the related contour equations are non-local, not only in the evolution system, but also in the implicit relation between the amplitude of the vorticity and the free interface. Among other extra difficulties, no maximum principles are available for the amplitude and the slopes of the interface in L^∞. We prove global in time existence results for medium size initial stable data in critical spaces. We also enhance previous methods by showing smoothing (instant analyticity), improving the medium size constant in 3D, together with sharp decay rates of analytic norms. The found technique is twofold, giving ill-posedness in unstable situations for very low regular solutions.

[**41**] Jian-Guo Liu and Robert M. Strain, *Global stability for solutions to the exponential PDE describing epitaxial growth*, Interfaces Free Bound. **21** (2019), no. 1, 61–86, http://arxiv.org/abs/1805.02246 , http://dx.doi.org/10.4171/IFB/417 , Zbl 07084773, MR 3951578.

Abstract: In this paper we prove the global existence, uniqueness, optimal large time decay rates, and uniform gain of analyticity for the exponential PDE $h_t = \Delta e^{-\Delta h}$ in the whole space $\mathbb{R}^d_x$. We assume the initial data is of medium size in the critical Wiener algebra $\Delta h \in A(\mathbb{R}^d)$. This exponential PDE was derived in (Krug, Dobbs, and Majaniemi in 1995) and more recently in (Marzuola and Weare 2013).

[**40**] Robert M. Strain and Maja Tasković, *Entropy dissipation estimates for the relativistic Landau equation, and applications*, J. Funct. Anal. **277** (2019), no. 4, 1139–1201, http://arxiv.org/abs/1806.08720 , http://dx.doi.org/10.1016/j.jfa.2019.04.007 , Zbl 07066836, MR 3959729.

Abstract: In this paper we study the Cauchy problem for the spatially homogeneous relativistic Landau equation with Coulomb interactions. Despite it's physical importance, this equation has not received a lot of mathematical attention we think due to the extreme complexity of the relativistic structure of the kernel of the collision operator. In this paper we first largely decompose the structure of the relativistic Landau collision operator. After that we prove the global Entropy dissipation estimate. Then we prove the propagation of any polynomial moment for a weak solution. Lastly we prove the existence of a true weak solution for a large class of initial data.

[**39**] Robert M. Strain and Zhenfu Wang, *Uniqueness of bounded solutions for the homogeneous relativistic Landau equation with Coulomb interactions*, Quart. Appl. Math. **78** (2019), 107–145, http://arxiv.org/abs/1903.05301, http://dx.doi.org/10.1090/qam/1545, Zbl 1427.82044, MR 4042221.

Abstract: We prove the uniqueness of weak solutions to the spatially homogeneous special relativistic Landau equation under the conditional assumption that the solution satisfies $(p^0)^7 F(t,p) \in L^1([0,T];L^\infty)$. The existence of standard weak solutions to the relativistic Landau equation has been shown recently in (Strain-Tasković 2019, 1806.08720).

[**38**] Neel Patel and Robert M. Strain, *Large time decay estimates for the Muskat equation*, Comm. Partial Differential Equations **42** (2017), no. 6, 977–999, http://arxiv.org/abs/1610.05271, http://dx.doi.org/10.1080/03605302.2017.1321661, Zbl 1378.35245, MR 3683311.

Abstract: We prove time decay of solutions to the Muskat equation in 2D and in 3D. We consider the norm $\|f\|_s(t)$. In this paper, for the 3D Muskat problem, given initial data $f_0 \in H^l(\mathbb{R}^2)$ for some $l \geq 3$ such that $\|f_0\|_1 < k_0$ for a constant $k_0 \approx 1/5$, we prove uniform in time bounds of $\|f\|_s(t)$ for $-d < s < l-1$ and assuming $\|f_0\|_\nu < \infty$ we prove time decay estimates of the form $\|f\|_s(t) \lesssim (1+t)^{-s+\nu}$ for $0 \leq s \leq l-1$ and $-d \leq \nu < s$. These large time decay rates are the same as the optimal rate for the linear Muskat equation. We also prove analogous results in 2D.

[**37**] Peter Constantin, Diego Córdoba, Francisco Gancedo, Luis Rodríguez-Piazza, and Robert M. Strain, *On the Muskat problem: global in time results in 2D and 3D*, Amer. J. Math. **138** (2016), no. 6, 1455–1494, http://arxiv.org/abs/1310.0953, http://dx.doi.org/10.1353/ajm.2016.0044, Zbl 1369.35053, MR 3595492.

Abstract: This paper considers the three dimensional Muskat problem in the stable regime. We obtain a conservation law which provides an L^2 maximum principle for the fluid interface. We also show global in time existence for strong and weak solutions with initial data controlled by explicit constants. Furthermore we refine the estimates from our prior paper to obtain global existence and uniqueness for strong solutions with larger initial data than we previously had in 2D. Finally we provide global in time results in critical spaces, giving solutions with bounded slope and time integrable bounded curvature.

[**36**] Jonathan Luk and Robert M. Strain, *Strichartz estimates and moment bounds for the relativistic Vlasov-Maxwell system*, Arch. Ration. Mech. Anal. **219** (2016), no. 1, 445–552, Paper combined from https://arxiv.org/abs/1406.0168 and https://arxiv.org/abs/1406.0169, http://dx.doi.org/10.1007/s00205-015-0899-1, Zbl 1337.35150, MR 3437855.

Abstract: We consider the relativistic Vlasov-Maxwell system with data of unrestricted size and without compact support in momentum space. In the two dimensional and the two-and-a-half dimensional cases, Glassey-Schaeffer proved (Commun Math Phys 185:257–284, 1997; Arch Ration Mech Anal 141:331–354, 1998; Arch Ration Mech Anal. 141:355–374, 1998) that for regular initial data with compact momentum support this system has unique global in time classical solutions. In this work we do not assume compact momentum support for the initial data and instead require only that the data have polynomial decay in momentum space. In the 2D and the $2\frac{1}{2}$D cases, we prove the global existence, uniqueness and regularity for solutions arising from this class of initial data. To this end we use Strichartz estimates and prove that suitable moments of the solution remain bounded. Moreover, we obtain a slight improvement of the temporal growth of the L^∞_x norms of the electromagnetic fields compared to (Commun Math Phys 185:257–284, 1997; Arch Ration Mech Anal 141:355–374, 1998). In the three dimensional case, we apply Strichartz estimates and moment bounds to show that a regular solution can be extended as long as $\|(p^0)^\theta f\|_{L^q_x L^1_p}$ remains bounded for $\theta > 2/q$ and $q \in (2, \infty]$. This improves previous results of Pallard (Indiana Univ Math J 54(5):1395–1409, 2005; Commun Math Sci 13(2):347–354, 2015).

[**35**] Robert M. Strain and Tak Kwong Wong, *Axisymmetric flow of ideal fluid moving in a narrow domain: a study of the axisymmetric hydrostatic Euler equations*, J. Differential Equations **260** (2016), no. 5, 4619–4656, http://arxiv.org/abs/1505.06281, http://dx.doi.org/10.1016/j.jde.2015.11.023, Zbl 1333.35170, MR 3437599.

Abstract: In this article we will introduce a new model to describe the leading order behavior of an ideal and axisymmetric fluid moving in a very narrow domain. After providing a formal derivation of the model, we will prove the well-posedness and provide a rigorous mathematical justification for the formal derivation under a new sign condition. Finally, a blowup result regarding this model will be discussed as well.

[**34**] Francisco Gancedo and Robert M. Strain, *Absence of splash singularities for surface quasi-geostrophic sharp fronts and the Muskat problem*, Proc. Natl. Acad. Sci. USA **111** (2014), no. 2, 635–639, http://arxiv.org/abs/1309.4023, http://dx.doi.org/10.1073/pnas.1320554111, Zbl 1355.76065, MR 3181769.

Abstract: In this paper for either the sharp front Surface Quasi-Geostrophic equation or the Muskat problem we rule out the ''splash singularity'' blow-up scenario; in other words we prove that the contours evolving from either of these systems can not intersect at a single point while the free boundary remains smooth. Splash singularities have been shown to hold for the free boundary incompressible Euler equation in the form of the water waves contour evolution problem (Castro A, et al. (2012) Proc Natl Acad Sci USA 109:733-738). Our result confirms the numerical simulations in (Córdoba D, et al. (2005) Proc Natl Acad Sci USA 102:5949-5952) where it is shown that the curvature blows up due to the contours collapsing at a point. Here we prove that maintaining control of the curvature will remove the possibility of pointwise interphase collapse. Another conclusion that we provide is a better understanding of the work (Córdoba D, Gancedo F (2010) Comm Math Phys 299:561-575) in which squirt singularities are ruled out; in this case a positive volume of fluid between the contours can not be ejected in finite time.

[**33**] Jonathan Luk and Robert M. Strain, *A new continuation criterion for the relativistic Vlasov-Maxwell system*, Comm. Math. Phys. **331** (2014), no. 3, 1005–1027, http://arxiv.org/abs/1406.0165, http://dx.doi.org/10.1007/s00220-014-2108-8, Zbl 1309.35174, MR 3248056.

Abstract: The global existence of solutions to the relativistic Vlasov-Maxwell system given sufficiently regular finite energy initial data is a longstanding open problem. The main result of Glassey-Strauss 1986 shows that a solution (f, E, B) remains C^1 as long as the momentum support of f remains bounded. Alternate proofs were later given by Bouchut-Golse-Pallard 2003 and Klainerman-Staffilani 2002. We show that only the boundedness of the momentum support of f *after projecting to any two dimensional plane* is needed for (f, E, B) to remain C^1.

[**32**] Vedran Sohinger and Robert M. Strain, *The Boltzmann equation, Besov spaces, and optimal time decay rates in* $\mathbb{R}^n_x$, Adv. Math. **261** (2014), 274–332, http://arxiv.org/abs/1206.0027, http://dx.doi.org/10.1016/j.aim.2014.04.012, Zbl 1293.35195, MR 3213301.

Abstract: We prove that k-th order derivatives of perturbative classical solutions to the hard and soft potential Boltzmann equation (without the angular cut-off assumption) in the whole space, $\mathbb{R}^n_x$ with $n \geq 3$, converge in large time to the global Maxwellian with the optimal decay rate of $t^{-\frac{1}{2}(k+r+\frac{n}{2}-\frac{n}{r})}$ in the $L^r_x(L^2_v)$-norm for any $2 \leq r \leq \infty$. These results hold for any $r \in (0, n/2]$ as long as initially $\|f_0\|_{\dot{B}^{-r,\infty}_2 L^2_v} < \infty$. In the hard potential case, we prove faster decay results in the sense that if $\|\mathbf{P}f_0\|_{\dot{B}^{-r,\infty}_2 L^2_v} < \infty$ and $\|\{\mathbf{I}-\mathbf{P}\}f_0\|_{\dot{B}^{-r+1,\infty}_2 L^2_v} < \infty$ for $r \in (n/2, (n+2)/2]$ then the solution decays the global Maxwellian in $L^2_v(L^2_x)$ with the optimal large time decay rate of $t^{-\frac{1}{2}r}$.

[**31**] Robert M. Strain and Seok-Bae Yun, *Spatially homogeneous Boltzmann equation for relativistic particles*, SIAM J. Math. Anal. **46** (2014), no. 1, 917–938, https://www.math.upenn.edu/~strain/preprints/92353.pdf, http://dx.doi.org/10.1137/130923531, Zbl 1321.35127, MR 3166961.

Abstract: The spatially homogeneous Boltzmann equation has been studied extensively in the Newtonian case, but not much is known for the special relativistic case. In this paper, we address several issues for the spatially homogeneous Boltzmann equation for relativistic particles. We first derive the relativistic version of the Povzner inequality. Using this, we study the Cauchy problem and investigate how the polynomial and exponential moments in L^1 are propagated. Several key differences between the relativistic and the Newtonian cases are confronted and discussed.

[**30**] Peter Constantin, Diego Córdoba, Francisco Gancedo, and Robert M. Strain, *On the global existence for the Muskat problem*, J. Eur. Math. Soc. (JEMS) **15** (2013), no. 1, 201–227, http://arxiv.org/abs/1007.3744, http://dx.doi.org/10.4171/JEMS/360, Zbl 1258.35002, MR 2998834.

Abstract: The Muskat problem models the dynamics of the interface between two incompressible immiscible fluids with different constant densities. In this work we prove three results. First we prove an $L^2(\mathbb{R})$ maximum principle, in the form of a new "log" conservation law which is satisfied by the equation for the interface. Our second result is a proof of global existence of Lipschitz continuous solutions for initial data that satisfy $\|f_0\|_{L^\infty} < \infty$ and $\|\partial_x f_0\|_{L^\infty} < 1$. We take advantage of the fact that the bound $\|\partial_x f_0\|_{L^\infty} < 1$ is propagated by solutions, which grants strong compactness properties in comparison to the log conservation law. Lastly, we prove a global existence result for unique strong solutions if the initial data is smaller than an explicitly computable constant, for instance $\|f\|_1 \leq 1/5$. Previous results of this sort used a small constant $\epsilon \ll 1$ which was not explicit.

[**29**] Seung-Yeal Ha, Eunhee Jeong, and Robert M. Strain, *Uniform* L^1*-stability of the relativistic Boltzmann equation near vacuum*, Commun. Pure Appl. Anal. **12** (2013),

no. 2, 1141–1161, https://www.math.upenn.edu/~strain/preprints/cpaa0494.pdf, http://dx.doi.org/10.3934/cpaa.2013.12.1141, Zbl 1267.35162, MR 2982812.

Abstract: We present the uniform L^1-stability estimate for the relativistic Boltzmann equation near vacuum. For this, we explicitly construct a relativistic counterpart of the nonlinear functional which is a linear combination of L^1-distance and a collision potential. This functional measures the L^1-distance between two continuous mild solutions. When the initial data is sufficiently small and decays exponentially fast, we show that the functional satisfies the uniform stability estimate leading to the uniform L^1-stability estimate with respect to initial data.

[**28**] Robert M. Strain and Keya Zhu, *The Vlasov-Poisson-Landau system in* $\mathbb{R}^3_x$, Arch. Ration. Mech. Anal. **210** (2013), no. 2, 615–671, http://arxiv.org/abs/1202.2471, http://dx.doi.org/10.1007/s00205-013-0658-0, Zbl 1294.35168, MR 3101794.

Abstract: For the Landau-Poisson system with Coulomb interaction in $\mathbb{R}^3_x$, we prove the global existence, uniqueness, and large time convergence rates to the Maxwellian equilibrium for solutions which start out sufficiently close.

[**27**] Hongjie Dong and Robert M. Strain, *On partial regularity of steady-state solutions to the 6D Navier-Stokes equations*, Indiana Univ. Math. J. **61** (2012), no. 6, 2211–2229, http://arxiv.org/abs/1101.5580, http://dx.doi.org/10.1512/iumj.2012.61.4765, Zbl 1286.35193, MR 3129108.

Abstract: Consider steady-state weak solutions to the incompressible Navier-Stokes equations in six spatial dimensions. We prove that the 2D Hausdorff measure of the set of singular points is equal to zero. This problem was mentioned in 1988 by Struwe, during his study of the five dimensional case.

[**26**] Renjun Duan and Robert M. Strain, *On the Full Dissipative Property of the Vlasov-Poisson-Boltzmann System*, Hyperbolic problems—theory, numerics and applications. Volume 2 (Tatsien Li and Song Jiang, eds.), Ser. Contemp. Appl. Math. CAM, vol. 17, World Scientific Publishing and Higher Education Press, Singapore, 2012, pp. 398–405, https://www.math.upenn.edu/~strain/preprints/2012DuanStrainHype.pdf, http://dx.doi.org/10/c74v, Zbl 1293.35339, MR 3050180.

Abstract: In this paper, we present a new approach of studying the full dissipative property of the Vlasov-Poisson-Boltzmann system over the whole space. The key part of this approach is to design the interactive functional to capture the dissipation of the system along the degenerate components. The developed approach is generally applicable to other relevant models arising from plasma physics both at the kinetic and fluid levels.

[**25**] Philip T. Gressman, Joachim Krieger, and Robert M. Strain, *A non-local inequality and global existence*, Adv. Math. **230** (2012), no. 2, 642–648, http://arxiv.org/abs/1202.4088, http://dx.doi.org/10.1016/j.aim.2012.02.017, Zbl 1248.35005, MR 2914961.

Abstract: In this article we prove a collection of new non-linear and non-local integral inequalities. We use these inequalities to deduce global existence of solutions to a non-local heat equation with a quadratic non-linearity for large radial monotonic positive initial conditions. Specifically, we include all $\alpha \in (0, \frac{74}{75})$.

[**24**] Yan Guo and Robert M. Strain, *Momentum regularity and stability of the relativistic Vlasov-Maxwell-Boltzmann system*, Comm. Math. Phys. **310** (2012), no. 3, 649–673, http://arxiv.org/abs/1012.1158, http://dx.doi.org/10.1007/s00220-012-1417-z, Zbl 1245.35130, MR 2891870.

Abstract: In the study of solutions to the relativistic Boltzmann equation, their regularity with respect to the momentum variables has been an outstanding question, even local in time, due to the initially unexpected growth in the post-collisional momentum variables which was discovered in 1991 by Glassey & Strauss. We establish momentum regularity within energy spaces via a new splitting technique and interplay between the Glassey-Strauss frame and the center of mass frame of the relativistic collision operator. In a periodic box, these new momentum regularity estimates lead to a proof of global existence of classical solutions to the two-species relativistic Vlasov-Maxwell-Boltzmann system for charged particles near Maxwellian with hard ball interaction.

[**23**] Joachim Krieger and Robert M. Strain, *Global solutions to a non-local diffusion equation with quadratic non-linearity*, Comm. Partial Differential Equations **37** (2012), no. 4, 647–689, http://arxiv.org/abs/1012.2890, http://dx.doi.org/10.1080/03605302.2011.643437, Zbl 1247.35087, MR 2901061.

Abstract: In this paper we prove the global in time well-posedness of the following non-local diffusion equation with $\alpha \in (0, 2/3)$: $\partial_t u = ((-\Delta)^{-1}u)\Delta u + \alpha u^2$. The initial condition u_0 is positive, radial, and non-increasing with $u_0 \in L^1 \cap L^{2+\delta}(\mathbb{R}^3)$ for some small $\delta > 0$. There is no size restriction on u_0. This model problem appears of interest due to its structural similarity with Landau's equation from plasma physics, and moreover its radically different behavior from the semi-linear Heat equation: $u_t = \Delta u + \alpha u^2$.

[**22**] Robert M. Strain, *Optimal time decay of the non cut-off Boltzmann equation in the whole space*, Kinet. Relat. Models **5** (2012), no. 3, 583–613, http://arxiv.org/abs/1011.5561, http://dx.doi.org/10.3934/krm.2012.5.583, Zbl 1383.76414, MR 2972454.

Abstract: In this paper we study the large-time behavior of perturbative classical solutions to the hard and soft potential Boltzmann equation without the angular cut-off assumption in the whole space $\mathbb{R}^n_x$ with $n \geq 3$. We use the existence theory of global in time nearby Maxwellian solutions from previous work. It has been a longstanding open problem to determine the large time decay rates for the soft potential Boltzmann equation in the whole space, with or without the angular cut-off assumption. For perturbative initial data, we prove that solutions converge to the global Maxwellian with the optimal large-time decay rate of $O(t^{-\frac{n}{2}+\frac{n}{2r}})$ in the $L^2_v(L^r_x)$-norm for any $2 \leq r \leq \infty$.

[**21**] Robert M. Strain and Keya Zhu, *Large-time decay of the soft potential relativistic Boltzmann equation in* $\mathbb{R}^3_x$, Kinet. Relat. Models **5** (2012), no. 2, 383–415, http://arxiv.org/abs/1106.1579, http://dx.doi.org/10.3934/krm.2012.5.383, Zbl 1247.76071, MR 2911100.

Abstract: For the relativistic Boltzmann equation in R^3_x, this work proves the global existence, uniqueness, positivity, and optimal time convergence rates to the relativistic Maxwellian for solutions which start out sufficiently close under the general physical soft potential assumption proposed in 1988.

[**20**] Renjun Duan and Robert M. Strain, *Optimal large-time behavior of the Vlasov-Maxwell-Boltzmann system in the whole space*, Comm. Pure Appl. Math. **64** (2011), no. 11, 1497–1546,

http://arxiv.org/abs/1006.3605, http://dx.doi.org/10.1002/cpa.20381, Zbl 1244.35010, MR 2832167.

Abstract: In this paper we study the large-time behavior of classical solutions to the two-species Vlasov-Maxwell-Boltzmann system in the whole space R^3_x. The existence of global in time nearby Maxwellian solutions is known from the work of Strain in 2006. However the asymptotic behavior of these solutions has been a challenging open problem. Building on our previous work on time decay for the simpler Vlasov-Poisson-Boltzmann system, we prove that these solutions converge to the global Maxwellian with the optimal decay rate of $O(t^{-\frac{3}{2}+\frac{3}{2r}})$ in $L^2_\xi(L^r_x)$-norm for any $2 \le r \le \infty$ if initial perturbation is smooth enough and decays in space-velocity fast enough at infinity. Moreover, some explicit rates for the electromagnetic field tending to zero are also provided.

[**19**] Renjun Duan and Robert M. Strain, *Optimal time decay of the Vlasov-Poisson-Boltzmann system in* $\mathbb{R}^3$, Arch. Ration. Mech. Anal. **199** (2011), no. 1, 291–328, http://arxiv.org/abs/0912.1742, http://dx.doi.org/10.1007/s00205-010-0318-6, Zbl 1232.35169, MR 2754344.

Abstract: The Vlasov-Poisson-Boltzmann System governs the time evolution of the distribution function for the dilute charged particles in the presence of a self-consistent electric potential force through the Poisson equation. In this paper, we are concerned with the rate of convergence of solutions to equilibrium for this system over R^3_x. It is shown that the electric field which is indeed responsible for the lowest-order part in the energy space reduces the speed of convergence and hence the dispersion of this system over the full space is slower than that of the Boltzmann equation without forces, where the exact difference between both power indices in the algebraic rates of convergence is $1/4$. For the proof, in the linearized case with a given non-homogeneous source, Fourier analysis is employed to obtain time-decay properties of the solution operator. In the nonlinear case, the combination of the linearized results and the nonlinear energy estimates with the help of the proper Lyapunov-type inequalities leads to the optimal time-decay rate of perturbed solutions under some conditions on initial data.

[**18**] Philip T. Gressman and Robert M. Strain, *Global classical solutions of the Boltzmann equation without angular cut-off*, J. Amer. Math. Soc. **24** (2011), no. 3, 771–847, http://arxiv.org/abs/1011.5441, http://dx.doi.org/10.1090/S0894-0347-2011-00697-8, Zbl 1248.35140, MR 2784329.

Abstract: This work proves the global stability of the Boltzmann equation (1872) with the physical collision kernels derived by Maxwell in 1866 for the full range of inverse-power intermolecular potentials, $r^{-(p-1)}$ with $p > 2$, for initial perturbations of the Maxwellian equilibrium states. We more generally cover collision kernels with parameters $s \in (0,1)$ and γ satisfying $\gamma > -n$ in arbitrary dimensions $\mathbb{T}^n \times \mathbb{R}^n$ with $n \ge 2$. Moreover, we prove rapid convergence as predicted by the celebrated Boltzmann H-theorem. When $\gamma \ge -2s$, we have exponential time decay to the Maxwellian equilibrium states. When $\gamma < -2s$, our solutions decay polynomially fast in time with any rate. These results are completely constructive. Additionally, we prove sharp constructive upper and lower bounds for the linearized collision operator in terms of a geometric fractional Sobolev norm; we thus observe that a spectral gap exists only when $\gamma \ge -2s$, as conjectured in Mouhot-Strain. It will be observed that this fundamental equation, derived by both Boltzmann and Maxwell, grants a basic example where a range of geometric fractional derivatives occur in a physical model of the natural world. Our

methods provide a new understanding of the grazing collisions in the Boltzmann theory.

[**17**] Philip T. Gressman and Robert M. Strain, *Sharp anisotropic estimates for the Boltzmann collision operator and its entropy production*, Adv. Math. **227** (2011), no. 6, 2349–2384, http://arxiv.org/abs/1007.1276, http://dx.doi.org/10.1016/j.aim.2011.05.005, Zbl 1234.35173, MR 2807092.

Abstract: This article provides sharp constructive upper and lower bound estimates for the Boltzmann collision operator with the full range of physical non cut-off collision kernels ($\gamma > -n$ and $s \in (0,1)$) in the trilinear $L^2(\mathbb{R}^n)$ energy $(\mathcal{Q}(g,f),f)$. These new estimates prove that, for a very general class of $g(v)$, the global diffusive behavior (on f) in the energy space is that of the geometric fractional derivative semi-norm identified in the linearized context in our earlier works. We further prove new global entropy production estimates with the same anisotropic semi-norm. This resolves the longstanding, widespread heuristic conjecture about the sharp diffusive nature of the non cut-off Boltzmann collision operator in the energy space $L^2(\mathbb{R}^n)$.

[**16**] Jared Speck and Robert M. Strain, *Hilbert expansion from the Boltzmann equation to relativistic fluids*, Comm. Math. Phys. **304** (2011), no. 1, 229–280, http://arxiv.org/abs/1009.5033, http://dx.doi.org/10.1007/s00220-011-1207-z, Zbl 1221.35271, MR 2793935.

Abstract: We study the local-in-time hydrodynamic limit of the relativistic Boltzmann equation using a Hilbert expansion. More specifically, we prove the existence of local solutions to the relativistic Boltzmann equation that are nearby the local relativistic Maxwellian constructed from a class of solutions to the relativistic Euler equations that includes a large subclass of near-constant, non-vacuum fluid states. In particular, for small Knudsen number, these solutions to the relativistic Boltzmann equation have dynamics that are effectively captured by corresponding solutions to the relativistic Euler equations.

[**15**] Robert M. Strain, *Coordinates in the relativistic Boltzmann theory*, Kinet. Relat. Models **4** (2011), no. 1, 345–359, http://arxiv.org/abs/1011.5093, http://dx.doi.org/10.3934/krm.2011.4.345, Zbl 05869610, MR 2765751.

Abstract: It is often the case in mathematical analysis that solving an open problem can be facilitated by finding a new set of coordinates which may illuminate the known difficulties. In this article, we illustrate how to derive an assortment coordinates in which to represent the relativistic Boltzmann collision operator. We show the equivalence between some known representations, and others which seem to be new. One of these representations has been used recently to solve several open problems.

[**14**] Philip T. Gressman and Robert M. Strain, *Global classical solutions of the Boltzmann equation with long-range interactions*, Proc. Natl. Acad. Sci. USA **107** (2010), no. 13, 5744–5749, https://www.math.upenn.edu/~strain/preprints/gsPNAS2010.pdf, http://dx.doi.org/10.1073/pnas.1001185107, Zbl 1205.82120, MR 2629879.

Abstract: This is a brief announcement of our recent proof of global existence and rapid decay to equilibrium of classical solutions to the Boltzmann equation without any angular cutoff, that is, for long-range interactions. We consider perturbations of the Maxwellian equilibrium states and include the physical cross-sections arising from an inverse-power intermolecular potential $r^{-(p-1)}$ with $p > 2$, and more generally. We

present here a mathematical framework for unique global in time solutions for all of these potentials. We consider it remarkable that this equation, derived by Boltzmann (1) in 1872 and Maxwell (2) in 1867, grants a basic example where a range of geometric fractional derivatives occur in a physical model of the natural world. Our methods provide a new understanding of the effects due to grazing collisions.

[**13**] Robert M. Strain, *Around the Boltzmann equation without angular cut-off*, Oberwolfach Rep. **7** (2010), no. 4, 3159–3236 (English), https://www.math.upenn.edu/~strain/preprints/rms2010ow.pdf, http://dx.doi.org/10.4171/owr/2010/54, Zbl 1235.00027.

Abstract: In this report, we will describe briefly several recent developments for the Boltzmann equation without the Grad angular cut-off assumption.

[**12**] Robert M. Strain, *Asymptotic stability of the relativistic Boltzmann equation for the soft potentials*, Comm. Math. Phys. **300** (2010), no. 2, 529–597, http://arxiv.org/abs/1003.4893, http://dx.doi.org/10.1007/s00220-010-1129-1, Zbl 1214.35072, MR 2728733.

Abstract: In this paper it is shown that unique solutions to the relativistic Boltzmann equation exist for all time and decay with any polynomial rate towards their steady state relativistic Maxwellian provided that the initial data starts out sufficiently close in L^∞_ℓ. If the initial data are continuous then so is the corresponding solution. We work in the case of a spatially periodic box. Conditions on the collision kernel are generic; this resolves the open question of global existence for the soft potentials.

[**11**] Robert M. Strain, *Global Newtonian limit for the relativistic Boltzmann equation near vacuum*, SIAM J. Math. Anal. **42** (2010), no. 4, 1568–1601, http://arxiv.org/abs/1004.5407, http://dx.doi.org/10.1137/090762695, Zbl 05894999, MR 2679588.

Abstract: We study the Cauchy Problem for the relativistic Boltzmann equation with near Vacuum initial data. Unique global in time mild solutions are obtained uniformly in the speed of light parameter $c \geq 1$. We furthermore prove that solutions to the relativistic Boltzmann equation converge to solutions of the Newtonian Boltzmann equation in the limit as $c \to \infty$ on arbitrary time intervals $[0, T]$, with convergence rate $1/c^{2-\epsilon}$ for any $\epsilon \in (0, 2)$. This may be the first proof of unique global in time validity of the Newtonian limit for a Kinetic equation.

[**10**] Chiun-Chuan Chen, Robert M. Strain, Tai-Peng Tsai, and Horng-Tzer Yau, *Lower bounds on the blow-up rate of the axisymmetric Navier-Stokes equations. II*, Comm. Partial Differential Equations **34** (2009), no. 1-3, 203–232, http://arxiv.org/abs/0709.4230, http://dx.doi.org/10.1080/03605300902793956, Zbl 1173.35095, MR 2512859.

Abstract: Consider axisymmetric strong solutions of the incompressible Navier-Stokes equations in $\mathbb{R}^3$ with non-trivial swirl. Let z denote the axis of symmetry and r measure the distance to the z-axis. Suppose the solution satisfies,for some $0 \leq \varepsilon \leq 1$ that $|v(x,t)| \leq C_* r^{-1+\varepsilon} |t|^{-\varepsilon/2}$ for $-T_0 \leq t < 0$ and a positive finite constant C_* which is allowed to be large, we then prove that v is regular at time zero.

[**9**] Chiun-Chuan Chen, Robert M. Strain, Horng-Tzer Yau, and Tai-Peng Tsai, *Lower bound on the blow-up rate of the axisymmetric Navier-Stokes equations*, Int. Math. Res. Not. IMRN (2008), no. 9, Art. ID rnn016, 31, http://arxiv.org/abs/math/0701796, http://dx.doi.org/10.1093/imrn/rnn016, Zbl 1154.35068, MR 2429247.

Abstract: Consider axisymmetric strong solutions of the incompressible Navier-Stokes equations in R^3 with non-trivial swirl. Such solutions are not known to be globally defined, but it is shown by Caffarelli-Kohn-Nirenberg in 1982 that they could only blow up on the axis of symmetry. Let z denote the axis of symmetry and r measure the distance to the z-axis. Suppose the solution satisfies the pointwise scale invariant bound $|v(x,t)| \leq C_*(r^2-t)^{-1/2}$ for $-T_0 \leq t < 0$ and a positive finite constant C_* which is allowed to be large, we then prove that v is regular at time zero.

[8] Robert M. Strain and Yan Guo, *Exponential decay for soft potentials near Maxwellian*, Arch. Ration. Mech. Anal. **187** (2008), no. 2, 287–339, https://www.math.upenn.edu/~strain/preprints/2005SGed.pdf, http://dx.doi.org/10.1007/s00205-007-0067-3, Zbl 1130.76069, MR 2366140.

Abstract: Consider both soft potentials with angular cutoff and Landau collision kernels in the Boltzmann theory inside a periodic box. We prove that any smooth perturbation near a given Maxwellian approaches to zero at the rate of $e^{-\lambda t^p}$ for some $\lambda > 0$ and $p \in (0,1)$. Our method is based on an unified energy estimate with appropriate exponential velocity weight. Our results extend the classical Caflisch 1980 result to the case of very soft potential and Coulomb interactions.

[7] Clément Mouhot and Robert M. Strain, *Spectral gap and coercivity estimates for linearized Boltzmann collision operators without angular cutoff*, J. Math. Pures Appl. (9) **87** (2007), no. 5, 515–535, http://arxiv.org/abs/math/0607495, http://dx.doi.org/10.1016/j.matpur.2007.03.003, Zbl 1388.76338, MR 2322149.

Abstract: In this paper we prove new constructive coercivity estimates for the Boltzmann collision operator without cutoff, that is for *long-range* interactions. In particular we give a generalized sufficient condition for the existence of a spectral gap which involves both the growth behavior of the collision kernel at large relative velocities and its singular behavior at grazing and frontal collisions. It provides in particular existence of a spectral gap and estimates on it for interactions deriving from the hard potentials $\phi(r) = r^{-(s-1)}$, $s \geq 5$ or the so-called moderately soft potentials $\phi(r) = r^{-(s-1)}$, $3 < s < 5$, (without angular cutoff). In particular this paper recovers (by constructive means), improves and extends previous results of Pao 1974. We also obtain constructive coercivity estimates for the Landau collision operator for the optimal coercivity norm pointed out in Guo 2002 and we formulate a conjecture about a unified necessary and sufficient condition for the existence of a spectral gap for Boltzmann and Landau linearized collision operators.

[6] Robert M. Strain, *On the linearized Balescu-Lenard equation*, Comm. Partial Differential Equations **32** (2007), no. 10-12, 1551–1586, http://arxiv.org/abs/math/0603490, http://dx.doi.org/10.1080/03605300601088609, Zbl 1128.76068, MR 2372479.

Abstract: The Balescu-Lenard equation from plasma physics is widely considered to include a highly accurate correction to Landau's fundamental collision operator. Yet so far it has seen very little mathematical study. We perform an extensive linearized analysis of this equation, which includes determining the asymptotic behavior of the new components of the linearized operator and establishing time decay rates for the linearized equation.

[5] Robert M. Strain, *Recent results on existence, uniqueness and asymptotic decay rates for collisional kinetic models*, Oberwolfach Rep. **3** (2006), no. 4, 3189–

3258 (English), https://www.math.upenn.edu/~strain/preprints/rms2006ow.pdf, http://dx.doi.org/10.4171/owr/2006/54, Zbl 1177.82057.

Abstract: We discuss recent work proving exponential time decay rates to equilibrium for Boltzmann equations such as the soft potentials, Landau's equation and the linearized Balescu-Lenard model. We also mention a proof of existence and unique- ness of solutions near Maxwellian to the Vlasov-Maxwell-Boltzmann system in the whole space. Some of these projects are joint work with Yan Guo.

[**4**] Robert M. Strain, *The Vlasov-Maxwell-Boltzmann system in the whole space*, Comm. Math. Phys. **268** (2006), no. 2, 543–567, http://arxiv.org/abs/math/0512002, http://dx.doi.org/10.1007/s00220-006-0109-y, Zbl 1129.35022, MR 2259206.

Abstract: The Vlasov-Maxwell-Boltzmann system is a fundamental model to describe the dynamics of dilute charged particles, where particles interact via collisions and through their self-consistent electromagnetic field. We prove the existence of global in time classical solutions to the Cauchy problem near Maxwellians.

[**3**] Robert M. Strain and Yan Guo, *Almost exponential decay near Maxwellian*, Comm. Partial Differential Equations **31** (2006), no. 1-3, 417–429, https://www.math.upenn.edu/~strain/preprints/2005SGaed.pdf, http://dx.doi.org/10.1080/03605300500361545, Zbl 1096.82010, MR 2209761.

Abstract: By direct interpolation of a family of smooth energy estimates for solutions near Maxwellian equilibrium and in a periodic box to several Boltzmann type equations, we show convergence to Maxwellian with any polynomial rate in time. Our results not only resolve the important open problem for both the Vlasov-Maxwell-Boltzmann system and the relativistic Landau-Maxwell system for charged particles, but also lead to a simpler alternative proof of recent decay results for soft potentials as well as the Coulombic interaction, with precise decay rate depending on the initial conditions.

[**2**] Robert M. Strain, *Some applications of an energy method in collisional Kinetic theory*, Ph.D. thesis, Brown University, 2005, (ProQuest Document ID 305028444), pp. 1–200, https://www.math.upenn.edu/~strain/preprints/2005PHDthesisS.pdf, MR 2707256.

Abstract: The collisional Kinetic Equations we study are all of the form

$$\partial_t F + v \cdot \nabla_x F + V(t,x) \cdot \nabla_v F = Q(F,F).$$

Here $F = F(t,x,v)$ is a probabilistic density function (of time $t \geq 0$, space $x \in \Omega$ and velocity $v \in \mathbb{R}^3$) for a particle taken chosen randomly from a gas or plasma. $V(t,x)$ is a field term which usually represents Maxwell's theory of electricity and magnetism, sometimes this term is neglected. $Q(F,F)$ is the collision operator which models the interaction between colliding particles. We consider both the Boltzmann and Landau collision operators. We prove existence, uniqueness and regularity of close to equilibrium solutions to the relativistic Landau-Maxwell system in the first part of this thesis. Our main tool is an energy method. In the second part, we prove arbitrarily high polynomial time decay rates to equilibrium for four kinetic equations. These are cutoff soft potential Boltzmann and Landau equations, but also the Vlasov-Maxwell-Boltzmann system and the relativistic Landau-Maxwell system. The main technique used here is interpolation. In the third part, we prove exponential decay for the cutoff soft potential Boltzmann and Landau equations. The main point here is to show that exponential

decay of the initial data is propagated by a solution. In the fourth and final part of this thesis, we write down a few important calculations in the relativistic Boltzmann theory which are scattered around the literature. We also calculate a few Lorentz transformations which maybe useful in relativistic transport theory. We use these calculations to comment about extending the results in this thesis to the relativistic Boltzmann equation.

[1] Robert M. Strain and Yan Guo, *Stability of the relativistic Maxwellian in a collisional plasma*, Comm. Math. Phys. **251** (2004), no. 2, 263–320, https://www.math.upenn.edu/~strain/preprints/2004SG.pdf, http://dx.doi.org/10.1007/s00220-004-1151-2, Zbl 1113.82070, MR 2100057.

Abstract: The relativistic Landau-Maxwell system is among the most fundamental and complete models for describing the dynamics of a dilute collisional plasma in which particles interact through Coulombic collisions and through their self-consistent electromagnetic field. We construct the first global in time classical solutions. Our solutions are constructed in a periodic box and near the relativistic Maxwellian, the Jüttner solution.

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