MATH 114 - EXTRA CREDIT PROBLEM # 4

Abstract. You can turn in this problem anytime before the final exam. Work on extra credit problems will be used to decide cases in which a course grade is in between two letter grades.

Galileo’s rolling ball experiment with friction

In class we discussed Galileo’s method for measuring the gravitation constant \( g \) by rolling a ball down a ramp which is placed at an angle \( \theta \) from being horizontal. The derivation we did neglected friction between the ball and the ramp as well as air resistance.

Try working out the equations of motion if you assume now that an additional force acts on the ball in the direction opposite to its motion down the ramp. You should assume that this force is proportional in magnitude to the speed of the ball. An additional constant \( c \) now appears in the theory which has to do with the relation of the frictional force to velocity.

Galileo’s original goal was to find \( g \) using experiments in which \( \theta \), the length of the ramp and the time the ball takes to get down the ramp are measured. The model we went over in class made it possible to find \( g \) from one such experiment if we neglect friction and wind resistance. In this naive model, the mass \( m \) of the ball did not appear in the final formula for the position of the ball at time \( t \). Does \( m \) appear in your improved theory? If \( m, c \) and \( g \) are not known at first, how many experiments would you need to determine \( g \) from the improved theory?

What does your improved model predict about the limiting velocity of the ball as a function of \( \theta, m, g \) and \( c \)?

Why is this problem similar to figuring out the velocity of a downhill ski racer? Do heavier downhill racers go faster?

If you have time, compare the predictions of the improved model to the model we discussed in class by rolling a ball down a real world ramp!