Dieting

This is perhaps the most practical differential equation ever, considering it could be used to figure out how much time it would take to lose a certain amount of weight if the number of calories ingested and the amount of calories burned working out and being active is constant. Yet the basal metabolic rate, also known as the resting metabolic rate, is higher the more one weighs, a reason it is easier to lose weight near the beginning of a diet plan instead of near the end.

Note: One must assume that spikes in metabolism, especially right after eating or working out, are ignored. The dieting plan is modeled as follows:

Differential equation:

\[ \frac{dB}{dt} = KB + m \]

where:

- \( B \) = body weight (kg)
- \( t \) = time
- \( K \) = constant
- \( m \) = \( i - b \)
- \( i \) = calories ingested
- \( b \) = calories burned
- \( \frac{dB}{dt} \) = change in weight

\[ \int \frac{dB}{kB + m} = \int dt \]

\[ \frac{d(KB+m)}{d(x)} = KB \]

\[ \frac{1}{K} \int \frac{d(KB+m)}{KB+m} = \int dt \]

\[ \frac{1}{K} \ln |KB+m| = t + C \]

\[ \ln |KB+m| = kt + KC \]

\[ KB+m = e^{kt+KC} \]

\[ KB+m = Ce^{kt} \]

\[ \beta = \frac{Ce^{kt} - m}{K} \]

At \( t = 0 \), \( P = P_0 \)

Solve for \( C \):

\[ Ce^{KC} = KB + m \]

\[ C = KB_0 + m \]

Inserting \( t + \) back,

\[ \beta = \frac{(KB_0 + m)e^{kt} - m}{K} \]

Note: \( K \) must be negative. To finally we need first week results. See page 2 for conclusion of problem.
Problem:
- Joe weighs 85 kg and he started a weight loss plan, not by eating 1600 calories of carbohydrates, 400 calories of protein, and 180 calories of fat. He also works out everyday, burning 800 calories, and has a job where he runs errands all day long, burning an additional 400 calories. If after seven days he weighs 83 kg, how long will it take Joe to reach his ideal weight of 75 kg?

Note: Calories must be converted to grams. There are 9 calories in a gram of fat, 4 calories in a gram of protein, and 4 calories in a gram of carbs. Hence, he ingests 220 grams of food, or .22 kg. We will estimate that in burning 1200 calories he burns 400 grams or .4 kg.

B_0 = 85 kg
r = 7
K = unknown
m = .22 - .40 = -.18 kg/day

\[
-3 = \left( KB_0 + m \right)e^{rt} - m
\]

\[
83 = \left( 85K - .18 \right)e^{.0013t} + .18
\]

Using the solve function on the calculator,

\[
K = -.0013
\]

To find out when he will weigh 75 kg,

\[
75 = (85(-.0013) -.18)e^{.0013t} + .18
\]

\[
t = 10 \left[ \frac{75 - .0013 - .18}{85(-.0013) - .18} \right]
\]

\[
t = 35.22
\]

\[\approx\] 36 days

Conclusion: It takes a little over 5 weeks for him to lose 10 kg. The calories he burns being active all day and exercising contribute for most of the weight loss, but his basal metabolic rate has a small yet significant effect.