MATH 210, PROBLEM SET 4

DUE IN LECTURE ON FRIDAY, MARCH 17.

1. Games with an infinite number of options

So far we have been discussing two-person zero sum games in which player I has some number n of options and player II has some number m of options. Can you think of a scenario in which player I has $n = 2$ options, but the options for player II are described by the choice of a real number r in the interval $0 \leq r \leq 1$? The payoff matrix for player I for such a game would have this form:

	Player II option r $0 \leq r \leq 1$
Player I option 1	$a_1(r)$
Player I option 2	$a_2(r)$

in which $a_1(r)$ and $a_2(r)$ are some functions of r. Assume that $a_1(r)$ and $a_2(r)$ are integrable functions of r. A mixed strategy for player I would play option #1 with probability p and option #2 with probability $1 - p$. A mixed strategy for player 2 would play option r with probability $q(r) \geq 0$, where we assume $q(r)$ is a continuous function of r such that

$$\int_{r=0}^{r=1} q(r)dr = 1$$

since this integral represents the probability of playing some strategy.

1. Explain why the integral

$$E(p,q) = \left(p \int_{r=0}^{r=1} a_1(r)q(r)dr\right) + \left((1-p)\int_{r=0}^{r=1} a_2(r)q(r)dr\right)$$

   is the expected payoff for player I of this choice of p and the function $r \to q(r)$.
2. Suppose that player 2 picks a choice of mixed strategy, represented by the continuous function q. Show that

$$\max_{0 \leq p \leq 1} E(p,q) = \max\{E(0,q), E(1,q)\} = \max(\int_{r=0}^{r=1} a_2(r)q(r)dr, \int_{r=0}^{r=1} a_1(r)q(r)dr).$$

   Why would player 2 want to find the function $q(r)$ which makes this as small as possible?
3. Show that all choices of $q(r)$ are optimal for player 2 if $a_1(r)$ and $a_2(r)$ are constant functions of r. Is this the only way that all $q(r)$ can be optimal?

2. The B.S. model with both a credibility and a lying benefit

In class we discussed the speaker versus listener game when the listener assigns a credibility benefit, as well as the case in which the speaker assigns a lying benefit. This problem is about the case in which these benefits both exist and are equal to a constant c in the range $0 < c < 100$. Here 100 stands, as usual, for the benefit to the speaker if a scandal is not believed to have happened by the listener.

4. Explain why the payoff matrix to the speaker is
$$A = \begin{pmatrix} -100 & -c & 100-2c \\ c & 0 & -c \\ 100+2c & c & -100 \end{pmatrix}$$
where as usual, the rows stand for the speaker's options of telling the truth, B.S.-ing or lying, and the columns stand for the listener thinking what was said is the truth, thinking it is B.S. and thinking it is a lie.
5. Assume $0 < c < 100$ from now on. Is there a dominant strategy?
6. By adding $\delta = 200$ to every entry of A, one gets a matrix A' with positive entries. Write down the linear programming problem involving $s = (s_1, s_2, s_3)$ which is associated to A'.
7. Show that the sum of the first and third columns of A' equals twice the middle column. Use this to show that every vertex $s = (s_1, s_2, s_3)$ of the linear programming problem must have at least one entry equal to 0.
8. Find all vertices of the linear programming problem for which two of s_1, s_2, s_3 equal 0, and calculate $f(s) = s_1 + s_2 + s_3$ for these vertices.
9. Find the remaining vertices, and determine the optimal vertices.
10. What are all the optimal strategies for the speaker in this game?

Extra Credit

These problems can be turned in at any time during the semester

EC1. With the notations of problems #1 - #3, how would you set up a linear programming problem appropriate for finding the optimal strategy $x^* = (p, 1-p)$ for player #1? Can you show that this linear programming problem is equivalent to finding an optimal strategy?

EC2. With the notations of problems #1 - #3, how would you try to set up a linear programming problem appropriate for finding the optimal strategy $y^* = q(r)$ for player #2? Can you show that this linear programming problem is equivalent to finding an optimal strategy?