

POST APOCALYPSE MATH 210

Probability
P. D. E.'s

1. If a medical test comes back positive, what are the odds it's correct?

Bayes Theorem.

2. How much social distancing is needed by different parts of the population?

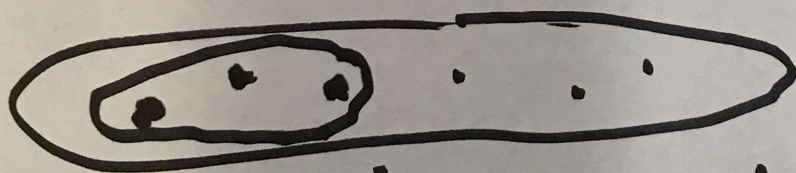
3. Can we come up with a "score" for each person's social distancing?
4. How do math models of epidemics work?
5. What is the physics behind person to person virus transmission?
"Cloud" versus "Bullet"?

Set up for Probability Theory

de Groot's book

S = sample space

= a set, whose elements
represent the possible
outcomes of an experiment



An event is a subset E of S

2^S = the set of all subsets
of S

Idea: We want to describe
which events $E \subseteq S$
we are interested in.

Def: A set σB of subsets E
of S is a σ -algebra if

1) S and \emptyset are in σB .

2) If E is in σB then
 $E^c = S - E$ is in σB .

3) If $\{E_i\}_{i=1}^{\infty}$ is a collection
of elements of σB then
 $\bigcup_{i=1}^{\infty} E_i$ is in σB .

Def: A probability function

P for a σ -algebra \mathcal{B}

assigns to each $E \in \mathcal{B}$

a number $P(E) \geq 0$ so

$$1) P(S) = 1$$

2) If $\{E_i\}_{i=1}^{\infty}$ are disjoint

events then

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

E.g.: $E \cup E^c = S$

$$P(E) + P(E^c) = P(S) = 1$$

$$P(E^c) = 1 - P(E)$$

Example 1:

$$S = \{a_1, \dots, a_m\} \text{ finite}$$

$$\mathcal{B} = 2^S = \text{all subsets of } S$$

$E = \text{event in } \mathcal{B}$

$$= \{a_1\} \cup \dots \cup \{a_\ell\}$$

for some distinct $a_1, \dots, a_\ell \in S$

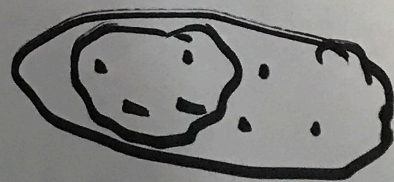
$$P(E) = P(\{a_1\}) + \dots + P(\{a_\ell\})$$

$$\text{If } P(\{a_i\}) = P(\{a_j\})$$


for $i \neq j$ then

$$P(E) = \ell \cdot P(\{a_1\}) = \ell/m = \frac{\#E}{\#S}$$

$$P(S) = 1 = m \cdot P(\{a_1\})$$



Example 2: $S = \mathbb{R}^n$

* $n=3$


Consider open subsets
 E of $S = \mathbb{R}^n$.

The Borel σ -algebra $\mathcal{B}_{\mathbb{R}^n}$
is the smallest σ -algebra
containing all open sets.

A probability function \mathbb{P} on $\mathcal{B}_{\mathbb{R}^n}$

has density function

$f: \mathbb{R}^n \rightarrow \mathbb{R}$ if f is continuous

and $\mathbb{P}(E) = \int_E f(t) dt$

E.g. " prob of finding a virus
molecule in E .