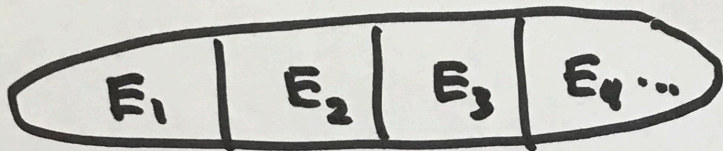


Updating Priors using Bayes Theorem

$$S = E_1 \cup E_2 \dots \cup E_n \quad \text{disjoint}$$



A = an observed event.

Suppose we know

$$P(E_1), \dots, P(E_n)$$

prior probabilities

$$P(A|E_1), \dots, P(A|E_n)$$

Bayes: Calculate posterior probabilities

$$\rightarrow P(E_1|A), \dots, P(E_n|A)$$

Use:
$$P(E_i|A) = \frac{P(A|E_i) P(E_i)}{\sum_{j=1}^n P(A|E_j) P(E_j)}$$

$$\sum_{j=1}^n P(A|E_j) P(E_j)$$

Example: "The signal + the noise"
by. N. Silver

9-11

E_1 = terrorists attack a NYC
skyscraper today

E_2 = they don't

A = a plane crashes into a NYC
skyscraper.

$$P(E_1 | A) = ? = \frac{P(A|E_1) P(E_1)}{P(A|E_1) P(E_1) + P(A|E_2) P(E_2)}$$

$$= \frac{1 \cdot (1/20000)}{1 \cdot (1/20000) + \frac{2}{25000} (1 - \frac{1}{20000})}$$

$$= 0.38$$

updated estimate
of $P(E_1)$ after
first attack.

Revised Priors

$$P(E_1) = 0.38 \quad \left(\begin{array}{c} \text{instead of} \\ \frac{1}{20000} \end{array} \right)$$

$$P(E_2) = 1 - P(E_1) = 0.62$$

A' = second ~~attack~~ ^{plane crash} happens

Assume $P(A'|E_1) = 1$, $P(A'|E_2) = \frac{2}{25000}$

Recalculate

$$P(E_1|A') = \frac{P(A'|E_1)P(E_1)}{P(A'|E_1)P(E_1) + P(A'|E_2)P(E_2)}$$

$$= \frac{1 \cdot (0.38)}{1 \cdot (0.38) + \frac{2}{25000} \cdot (0.62)}$$

$$\approx 0.9999..$$

Forgeries

E_1 = a forger is trying to pass off a rare manuscript

E_2 = there is not one

A = a rare manuscript appears

$P(E_1) = 10^{-3}$ initial estimate

$$\begin{aligned} P(E_1|A) &= \frac{P(A|E_1) \cdot P(E_1)}{P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2)} \\ &= \frac{1 \cdot 10^{-3}}{1 \cdot 10^{-3} + 5 \cdot 10^{-4} \cdot (1 - 10^{-3})} \approx \frac{2}{3} \end{aligned}$$

then update $P(E_1)$ to $P(E_1) = \frac{2}{3}$

A' = a second copy appears!

If $P(A'|E_1) = P(A|E_1)$, $P(A'|E_2) = P(A|E_2)$

then $P(E_1|A') \sim 1$.