1. (a) Let $K$ be a field of characteristic $p$ and let $a \in F$. Show that the polynomial $x^p - a \in F[x]$ is either irreducible or is the $p$-th power of a linear polynomial.

(b) Let $P$ be a field of characteristic $p > 0$ and let $F = P(t)$ be the field of rational functions with coefficients in $P$. Show that the polynomial $x^p - t \in F[x]$ is irreducible.

2. Let $\Phi_n(x)$ be the $n$-th cyclotomic polynomial. Show that for every $d < n$ with $d|n$ the product $(x^d - 1)\Phi_n(x)$ divides $x^n - 1$ in the polynomial ring $\mathbb{Z}[x]$.

3. Consider the tower of finite fields

$$F_p \subset F_p^{2^1} \subset F_p^{2^2} \subset F_p^{2^3} \subset \cdots.$$ 

Let $F_{p^{\infty}} = \bigcup_{n=1}^{\infty} F_{p^n}$ with the natural addition and multiplication. Show that $F_{p^{\infty}}$ is algebraically closed.