(1) Show that an open subset of $\mathbb{R}$ is a countable union of (open) intervals. What can you say about closed subsets of $\mathbb{R}$.

(2) Are the closures and interiors of connected sets always connected?

(3) Examine the question when all closed balls $\{q|d(p, q) \leq r\}$ around $p$ of radius $r$ in a metric space $X$ are compact for all $p$ and $r$. Is this true for all metric spaces?, Complete metric spaces?, $X = \mathbb{R}^n$? Is the closure of an open ball $\{q|d(p, q) < r\}$ the closed ball $\{q|d(p, q) \leq r\}$? Give example and counter examples.

(4) Let $\{a_n\}$ be a sequence of real numbers that converge to $\alpha$. Show that the averages $A_n = (a_1 + \ldots a_n)/n$ also converge to $\alpha$.

(5) (a) Compute $\lim_{n \to \infty} \frac{a^n}{b^n}$ where $a, b \in \mathbb{R}$.

(b) Compute $\lim_{n \to \infty} \frac{a^n}{n!}$ where $a \in \mathbb{R}$.

(c) Compute $\lim_{n \to \infty} (\sqrt[n]{n} - 1)^n$.

(6) Define a sequence via $x_{n+1} = \sqrt{2 + \sqrt{x_n}}$ with a prescribed positive value of $x_1$. Show it converges. Can you determine its limit? An upper bound for the limit?

(7) Determine if the following series converges or diverges:

$$1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{7} - \frac{1}{8} \ldots$$

(the sign pattern is $++--++--++\ldots$).