(1) Show that $\sin(x)$ and $\cos(x)$ are continuous at all points. Are they uniformly continuous? Use the definitions in terms of angles and you can use any trigonometric formula.

(2) Let $X = \mathbb{R} - \{0\}$ and $f : X \rightarrow \mathbb{R}$ given by $f(x) = 1/x^2$. Show that $f$ is continuous but not uniformly continuous.

(3) Show that $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is continuous at $x_0$ iff $\lim_{x \to x_0} |f(x) - f(x_0)| = 0$.

(4) Let $f : [a, b] \rightarrow \mathbb{R}^n$ be a function. Show that $f$ is continuous if and only if the graph
\[ \{(x, f(x)) \in \mathbb{R}^{n+1} \mid x \in [a, b]\} \]
is a compact subset of $\mathbb{R}^{n+1}$.

(5) Show that the image of a bounded set under a uniformly continuous function $f : E \rightarrow \mathbb{R}$, $E \subset \mathbb{R}$, is bounded. Give an example that this is not true if $f$ is continuous but not uniformly continuous.

(6) Let $f : [a, b] \rightarrow [a, b]$ be continuous. Show that $f$ has at least one fixed point, i.e., a point $c \in [a, b]$ so that $f(c) = c$.

(7) Show that a (complex) power series $f(z) = \sum a_k(z - z_0)^k$ is continuous within its radius of convergence, i.e. for all points in the open disc $|z - z_0| < R$. Give an example that it may not be uniformly continuous.

(8) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function with $f(x + y) = f(x) + f(y)$ for all $x, y$. Show that there is a constant $c$ such that $f(x) = cx$. Is this true if we do not assume $f$ is continuous?