Math 601 Spring 2015

Homework 10

Due Thursday April 16 at 1pm in Chris Hays mailbox

(1) Show there is no homotopy equivalence $f\colon \mathbb{CP}^{2n} \to \mathbb{CP}^{2n}$ which reverses orientation.

(2) If M, N are two compact oriented n-dimensional manifolds with fundamental classes $[M]$ and $[N]$, then we can define a degree of a continuous map $f\colon M \to N$ as the integer d with $f_*([M]) = d[N]$.
   (a) Show that for a compact orientable manifold M^n there exists a degree 1 map $f\colon M \to \mathbb{S}^n$.
   (b) If $f\colon \mathbb{S}^n \to M$ has non-zero degree, show that $H^*(M, \mathsf{Q}) \simeq H^*(\mathbb{S}^n, \mathsf{Q})$. Is this true for $\mathbb{Z}$ coefficients as well?

(3) If M, N are two compact oriented n-dimensional manifolds and $f\colon M \to N$ a d-fold cover, show that $deg(f) = d$.

(4) Return to the homology groups, and cohomology groups, of a connected sum $M\#N$ discussed in a previous assignment. You can now discuss the various possibilities for $H_i(M\#N, \mathbb{Z})$ also in the case of $i = n-1$ and $i = n$ (a case you could not deal with before). What can you say about the relationship of the ring structure?

(5) (a) Show that if M is $\mathbb{Z}$ orientable, it is R orientable for all rings R.
   (b) Show that if M is $\mathbb{Z}_p$ orientable for some prime $p \neq 2$, it is $\mathbb{Z}$ orientable.