Math 601 Spring 2015

Homework 11

Due Thursday April 23 at 1pm in Chris Hays mailbox

(1) Let $M$ be a 3-dimensional non-orientable topological manifold. Show that $H_1(M, \mathbb{Z})$ is infinite.

(2) Let $M$ be an $n$-dimensional non-orientable topological manifold. Show that $H_c^n(M, \mathbb{Z}) = \mathbb{Z}_2$.

(3) Show that $S^2 \times S^2$, $CP^2 \# CP^2$ and $CP^2 \# CP^2$ all have the same cohomology groups, but different homotopy type by determining the intersection form.

(4) Show that a compact orientable manifold of dimension $4n + 2$ has even Euler characteristic. What can you say for other dimensions?

(5) Let $X$ be a $4k$-dimensional compact orientable topological manifold. Show that the signature has the same parity as the Euler characteristic.