(1) Let $\sigma: E \to B$ be a covering with $E$ locally path connected, and deck group $D$. Show that $D$ operates properly discontinuously on $E$. (Do you need that $E$ is locally path connected?)

(2) Let $E \to B$ be a covering and assume that $B$ is a differentiable manifold.
   (a) Show that $E$ is a differentiable manifold such that $p$ is smooth. (I relieve you of the duty to show that $E$ is also second countable, which is more involved).
   (b) Show that the deck group of $E$ acts smoothly on $E$.
   (c) Show that any cover of a Lie group is a Lie group.

(3) Let $p: E \to G$ be a covering of topological groups (or Lie groups) with $E$ path connected. This includes by definition that $p$ is a group homomorphism as well.
   (a) Show that $\ker p$ is a discrete subgroup of $Z(E)$, the center of $E$.
   (b) Show that $p$ is a normal (regular) cover. What is the deck group?
   (c) Show conversely that if $G$ is a topological group and $\Gamma \subset Z(G)$ a discrete subgroup, then the projection $G \to G/\Gamma$ is a covering.

(4) Let $\mathbb{H}$ be the algebra of quaternions with norm $|q|^2 = a^2 + b^2 + c^2 + d^2$ for $q = a + bi + cj + dk \in \mathbb{H}$.
   (a) Show that $S^3 = \{q \in \mathbb{H} \mid |q| = 1\}$ is a Lie group (sometimes also called $Sp(1)$).
   (b) Show that $\sigma: S^3 \to SO(3)$ is a 2-fold cover, where $\sigma(q)(v) = qvq^{-1}$ for $v \in \text{Im} \mathbb{H}$ (first make sense out of this definition).
   (c) Show that $\sigma: S^3 \times S^3 \to SO(4)$ is a 2-fold cover, where $\sigma((q_1, q_2))(v) = q_1 v q_2^{-1}$ for $v \in \mathbb{H}$.

(5) Write the Klein bottle as the union of 2 triangles (with appropriate identifications of the sides). Compute its simplicial homology, both with with $\mathbb{Z}$ coefficients as well as $\mathbb{Z}_p$ coefficients for $p$ a prime. Notice that this is strictly speaking not a triangulation (why not?) but the formulas for computing homology is the same.

(6) (Extra Credit) Show that $SO(4)$ and $S^3 \times SO(3)$ are Lie groups with the same fundamental group. Furthermore show that they are diffeomorphic as manifolds but not isomorphic as Lie groups (or even as groups).