Math 601 Spring 2015

Homework 7

Due Thursday March 26 at 1pm in Chris Hays mailbox

(1) Let X be the space obtained by attaching a ball B^{n+1} to $\mathbb{R}P^n$ where the attaching map $f : \partial B \to \mathbb{R}P^n$ is a composition of a map of degree d from ∂B to the n-sphere S^n with the standard double covering from S^n to $\mathbb{R}P^n$. Compute the homology groups of X with coefficients in both $\mathbb{Z}$ and $\mathbb{Z}_2$.

(2) Let Y ba a finite simplicial complex and $X \to Y$ an n-fold cover. Show that the Euler characteristic of X is n times that of Y. Illustrate with the classification of orientable and non-orientable compact surfaces.

(3) Show that every continuous map $f\colon \mathbb{S}^2 \to \mathbb{S}^1 \times \mathbb{S}^1$ is null homotopic. On the other hand, show that the quotient map $g\colon \mathbb{S}^1 \times \mathbb{S}^1 \to \mathbb{S}^2$ which collapses $\mathbb{S}^1 \vee \mathbb{S}^1$ to a point is not null homotopic.

(4) A map $f\colon \mathbb{S}^n \to \mathbb{S}^n$ satisfying $f(x) = f(-x)$ for all x is called an even map. Show that an even map must have even degree, and that the degree is zero when n is even.

(5) (a) Assume that X is a compact, connected topological space with $\pi_1(X)$ equal to the symmetric group S_3, $H_2(X, \mathbb{Z}) = \mathbb{Z} \oplus \mathbb{Z}_6$, and $H_3(X, \mathbb{Z}) = \mathbb{Z}^2 \oplus \mathbb{Z}_7$. Compute $H_*(X, \mathbb{Z}_2)$ and $H_*(X, Z_{14})$ for $* \leq 3$.

(b) Compute the homology groups of $X \times K$, where X is the space in part (a) and K the Klein bottle.