(1) Let $X$ be the space obtained by attaching a ball $B^{n+1}$ to $\mathbb{R}P^n$ where the attaching map $f : \partial B \to \mathbb{R}P^n$ is a composition of a map of degree $d$ from $\partial B$ to the $n$-sphere $S^n$ with the standard double covering from $S^n$ to $\mathbb{R}P^n$. Compute the homology groups of $X$ with coefficients in both $\mathbb{Z}$ and $\mathbb{Z}_2$.

(2) Let $Y$ be a finite simplicial complex and $X \to Y$ an $n$-fold cover. Show that the Euler characteristic of $X$ is $n$ times that of $Y$. Illustrate with the classification of orientable and non-orientable compact surfaces.

(3) Show that every continuous map $f : S^2 \to S^1 \times S^1$ is null homotopic. On the other hand, show that the quotient map $g : S^1 \times S^1 \to S^2$ which collapses $S^1 \vee S^1$ to a point is not null homotopic.

(4) A map $f : S^n \to S^n$ satisfying $f(x) = f(-x)$ for all $x$ is called an even map. Show that an even map must have even degree, and that the degree is zero when $n$ is even.

(5) (a) Assume that $X$ is a compact, connected topological space with $\pi_1(X)$ equal to the symmetric group $S_3$, $H_2(X, \mathbb{Z}) = \mathbb{Z} \oplus \mathbb{Z}_6$, and $H_3(X, \mathbb{Z}) = \mathbb{Z}^2 \oplus \mathbb{Z}_7$. Compute $H_*(X, \mathbb{Z}_2)$ and $H_*(X, \mathbb{Z}_{14})$ for $* \leq 3$.

(b) Compute the homology groups of $X \times K$, where $X$ is the space in part (a) and $K$ the Klein bottle.