ASSIGNMENT # 10

MATH 660 , DIFFERENTIAL GEOMETRY

(1) Let $N^k \subset M^n$ be a compact submanifold and

$$\nu_\epsilon = \{v \in T_pM \mid p \in N, \text{ and } v \perp T_pN \text{ with } |v| \leq \epsilon\}.$$

Show that there exists an $\epsilon > 0$ such that $\exp^\perp \colon \nu_\epsilon \to M$ (where $\exp^\perp(v) = \exp_p(v)$ for $v \in T_pM$) is a diffeomorphism onto its image. In this case $T_\epsilon(N) = \exp^\perp(\nu_\epsilon)$ is called an ϵ tubular neighborhood of N.

(2) $N^k \subset M^n$ as above. Prove an analogue of the Gauss Lemma: The geodesic $\exp^\perp(tv)$ meets $\partial T_\delta(N)$ orthogonally for all $\delta \leq \epsilon$.

(3) $N^k \subset M^n$ as above. If $\gamma(t) = \exp^\perp(tv),\ t \leq a$ is a geodesic in a tubular neighborhood, show that γ is the shortest connection from N to $\gamma(a)$.

(4) Let $N^k \subset R^N$ be a submanifold in Euclidean space. What are the focal points of N along a geodesic orthogonal to N?

(5) What is the analogue of the index form for curves starting at $p \in M$ and ending at $N \subset M$. Show that it is positive definite if there are no focal points.