Math 114. Fall 2014. HW 4. Due Oct 22nd Wednesday

Instructions for written homework.

- You are encouraged to work with others on these problems. You are expected to write the solutions yourself.
- Your solutions should be legible and well organized. **Graders will deduct points for solutions that are difficult to read, or are disorganized.** For the benefit of the grader, please turn in solutions to problems in the assigned order, i.e. #1, then #2, then #3, etc.
- Staple your pages together. Do not turn in notebook paper with tattered edges. **Homework that is unstapled or is lacking a name will not be graded.**

Problem 1 (Fall 2011)**.** Which of the following limits exist?

a.

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 - y^2}$$

b.

$$\lim_{(x,y)\to(0,0)} \frac{x - y}{x^2 + y^2}$$

c.

$$\lim_{(x,y)\to(0,0)} \frac{x + y}{\sqrt{x^2 + y^2}}$$

for $0 \leq t \leq 2$

Problem 2. Does the following limit exist

$$\lim_{(x,y)\to(0,0)} \frac{(x^2 + y^2)\sin(x^2 + y^2)}{x^4 + y^4}?$$

If yes, find the limit.

Problem 3 (Fall 2010)**.** The function $z = f(x, y)$ is given implicitly by the equation $z^3 + z = x^2 + y^2$. Note that when $x = 1$ and $y = 1$, $z = 1$ as well. Compute $\frac{\partial f}{\partial x}(1.1)$

Problem 4 (Fall 2010)**.** Consider the surface $z = x^2 + x + 2y^2$. At what point (x_0, y_0, z_0) is the tangent plane parallel to the plane $x + 4y + z = 0$? What is the z coordinate of that point ?

Problem 5 (Sprint 2008). Let f be the function

$$f(x, y) = \ln(x + y)$$

for every $(x, y) \in \mathbb{R}^2$ and $x + y > 0$. A unit vector in $\mathbb{R}^2$ is a vector of length 1. What is the maximum value of the directional derivative $D_{\vec{u}}(f)$ of f at the point $(x, y) = (2, -1)$ as $\vec{u}$ ranges over all unit vectors in $\mathbb{R}^2$.

Problem 6 (Spring 2008). Find the equation of the tangent plane to the surface

$$4x^4 + 2y^4 + z^4 = 22$$

at the point $(1, 1, 2)$.

Problem 7 (Fall 2008). Let $T(x, y) = x^2 + y^2 - x - y$ be the temperature of at the point (x, y) in the plane. A lizard sitting at the point $(1, 3)$ wants to increase his surrounding temperature as quickly as possible. In which direction should he move?

Problem 8 (Spring 2013). Let

$$f(x, y, z) = \ln(x^2 + y^2) - z^3.$$

Using the linearization of f at $(-1, 1, 1)$ estimate the value of $f(-0.9, 1.2, 1.1)$. (Your final answer can contain $\ln 2$.)

Problem 9 (Spring 2013). Find the local minimum of the following function

$$f(x, y) = x^3 - 3xy + y^2$$

Problem 10 (Fall 2012). Find all the critical points of the function $h(x, y) = 2x\sin(y) + y^2 - x^2$ and determine which is a local maximum, which is a local minimum and which is saddle point.