HW4 MATH 420

Here are the exercise from the book:

Section 2.2   # 10, 11, 15, 20

Section 2.3   # 3, 5, 8, 9

Section 2.4   # 5, 6, 7

Here are the ones not from the book:

Problem 1: Find the general solutions for

(0.1)
$$\begin{cases} \frac{dx}{dt} = 3x - y^{1007} \\ \frac{dy}{dt} = -2y \end{cases}$$

Problem 2.

(1) Let a be a constant, $f_1(t)$ and $f_2(t)$ be functions of t. Prove that if $X_1(t)$ is a solution for

$$\frac{dx}{dt} = ax + f_1(t)$$

and $X_2(t)$ is a solution for

$$\frac{dx}{dt} = ax + f_2(t)$$

Then $X_1(t) + X_2(t)$ is a solution for

$$\frac{dx}{dt} = ax + f_1(t) + f_2(t)$$

(2) Can you generalize this result? Let $1 \le i \le n$, if $X_i(t)$ be a solution for

$$\frac{dx}{dt} = ax + f_i(t)$$

Then $X(t) = \sum_{i=1}^{n} X_i(t)$ is a solution for

$$\frac{dx}{dt} = ax + \sum_{i=1}^{n} f_i(t).$$

(3) Use the above result to find the solutions for the following system:

(0.2)
$$\begin{cases} \frac{dx}{dt} = 3x + y^{\alpha} + y^{\beta} + y^{\gamma}, \\ \frac{dy}{dt} = -y \end{cases}$$

where $\alpha = $ the year you came to Penn, $\beta = $ the year you will leave Penn, $\gamma = $ the year Penn was founded.