1. Use the first few terms from the Maclaurin series expansion to find the limit:

\[
\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6} x^3}{x^5} = \lim_{x \to 0} \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} - x + \frac{1}{6} x^3
\]

\[
\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} - x + \frac{1}{6} x^3 = \lim_{x \to 0} \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} + O(x^7) - x + \frac{x^3}{6}}{x^5}
\]

\[
= \frac{1}{5!} = \frac{1}{120}
\]

Here ‘big O’ notation \(O(x^7)\) means \(\sim Cx^7\) with \(C\) a constant.

2. Use the Maclaurin series and (limiting) comparison test to determine whether the series converges:

\[
\sum_{n=1}^{\infty} \frac{(3^{\frac{1}{n}} - 1)}{\ln(\ln n)}
\]

Hint: consider in steps,

\[
\sum_{n=1}^{\infty} (3^{\frac{1}{n}} - 1), \quad \sum_{n=1}^{\infty} \frac{(3^{\frac{1}{n}} - 1)}{\ln n}
\]

then the actual problem.
We did in class

\[ 3^x = e^{x \ln 3} = \sum_{m=0}^{\infty} (x \ln 3)^m = 1 + x \ln 3 + O(x^2) \]

Check the theorem for estimation for the remainder of a Taylor series for the 'big O' estimation. Therefore from the original series, plug in \( x = \frac{1}{n} \) from above

\[ a_n = \frac{3^{\frac{1}{n}} - 1}{\ln(\ln n)} = \frac{1 + \frac{1}{n} \ln 3 + O\left(\frac{1}{n^2}\right)}{\ln(\ln n)} > \frac{\ln 3}{n \ln n} \]

And the smaller series \( \sum \frac{\ln 3}{n \ln n} \) diverges by integration test so the original series diverges also.

If you can work out this problem all on your own, you’ll feel more acquainted with how to use comparison test.

3. Solve the differential equation:(2pt)

\[ y' = y^2 \sin x, \quad y(0) := y|_{x=0} = 2. \]

\[ \frac{1}{y^2} dy = \sin x dx \]

integrate on both sides,

\[ -\frac{1}{y} = - \cos x + C \]

Plug in \( y(0) = 2 \), get \( C = \frac{1}{2} \), etc.