1. Prove that a subgroup $H$ of $G$ is normal if and only if $Hg = gH$ for any $g \in G$. Here $gH = \{gh|h \in H\}$ and $Hg = \{hg|h \in H\}$.

2. Let $H$ be a subgroup of $G$ with $|G/H| = 2$. Prove that $H$ is a normal subgroup.

3. Prove that a normal subgroup $H$ of $G$ is the union of some conjugacy classes in $G$.

4. The 2-cycles $(i_1, i_2)$ in symmetric group $S_n$ are called transpositions. Prove that every element $x \in S_n$ can be written as a product of transpositions. (Hint: use induction on $|S|$ where $S = \{i \in \{1, \cdots, n\} | x(i) \neq i\}$.)

5. In this question, you will classify all the normal subgroups of $S_4$.
   (a) How many conjugacy classes are there in $S_4$?
   (b) List all the elements in each conjugacy class.
   (c) Find possible subsets $G$ of $S_4$ such that
      i. $G$ contains identity,
      ii. $G$ is the union of some conjugacy classes,
      iii. $|G|$ divides $|S_4|$.
   (d) Find all normal subgroups of $S_4$ (based on problem 3 and problem 4).

6. Prove
   (a) Any subgroup of a cyclic group $C_n$ is still a cyclic group.
   (b) Any subgroup of dihedral group $D_n$ is either a cyclic group or a dihedral group.

7. Let $y_1, y_2 \in O(2)$ be two reflections about lines $l_1, l_2$. Assume the angle between $l_1$ and $l_2$ is $\theta$. Find all the possible compositions $y_1y_2$.

8. Find all the normal subgroups of $D_4$. (Hint: use the procedure described in problem 5.)