1. Prove the semidirect product $H \rtimes \varphi K$ defined in the class gives a group structure on $H \times K$. Let $H$ and $K$ be two groups and $\varphi : K \to \text{Aut}(H)$ be a group homomorphism. Define a binary operation on $H \times K$ by $(h,k)(h',k') = (h\varphi(k)(h'),kk')$. Check this binary operation gives a group structure.

2. Artin, Chapter 7, 8.6
Let $G$ be a group of order 55

(a) Prove that $G$ is generated by elements $x$ and $y$ with relations $x^{11} = 1, y^{5} = 1$ and $yx y^{-1} = x^{r}$.

(b) Find the possible values of $r$.

(c) Prove there are two isomorphism classes of $G$. 