

HW 5. Problem 6.

(1) check K is closed under multiplication and inverse.

for example:

$$(12)(34)(23)(14) = (13)(24)$$

$$((12)(34))^2 = (1)$$

$$(2) |A_4/K| = \frac{|A_4|}{|K|} = \frac{12}{4} = 3$$

$$\text{So } A_4/K \cong C_3.$$

(3). A_4/K has character table.

	(1)	a	a^2
χ_1	1	1	1
χ_2	1	w	w^2
χ_3	1	w^2	w

where

$$\alpha = (123) \in K.$$

$$w = e^{\frac{2\pi i}{3}} = \frac{-1 + \sqrt{3}i}{2}$$

So A_4 has the liftings

	$\widetilde{\chi}_1$	$\widetilde{\chi}_2$	$\widetilde{\chi}_3$	$A_k \rightarrow A_k/K \rightarrow G$
	(1)	(12)(34)	(123)	(132)
$\widetilde{\chi}_1$	1	1	1	1
$\widetilde{\chi}_2$	1	1	ω	ω^2
$\widetilde{\chi}_3$	1	1	ω^2	ω
χ_x	a_1	a_2	a_3	a_4

We get χ_x by orthogonal relations.

$$1^2 + 1^2 + 1^2 + a_1^2 = 12 \Rightarrow a_1 = 3.$$

$$a_2 \cdot a_1 + 1 + 1 + 1 = 0 \Rightarrow a_2 = -1.$$

$$a_3 \cdot a_1 + 1 + \omega + \omega^2 = 0 \Rightarrow a_3 = 0$$

$$a_4 \cdot a_1 + 1 + \omega^2 + \omega = 0 \Rightarrow a_4 = 0.$$

$$\chi_x \quad 3 \quad -1 \quad 0 \quad 0.$$