# Math 371 Homework#4

Due on 2/25 at the beginning of Lecture

1. Let $R\colon G \to \mathrm{GL}(2, \mathbb{C})$ be a matrix representation of finite group. Prove that $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ can not appear in the image of $R$.

2. Let $e_1 \cdots e_n$ be the standard basis of $\mathbb{C}^n$. Define the linear operation $\rho$ of $S_n$ on vector space $\mathbb{C}^n$ by $\sigma e_i = e_{\sigma(i)}$. Prove that $\rho$ is not irreducible and write $\mathbb{C}^n$ as direct sum of two nontrivial invariant subspaces.

3. Let $V$ be an irreducible representation of $G$ and $\dim V \geq 2$. Prove that

$$\sum_{g \in G} gv = 0$$

for all $v \in V$.