Math 371 Homework#10

Due on 4/19

1. **Artin Chapter 11, 8.3** Prove that $\mathbb{F}_2[x]/(x^3+x+1)$ is a field, but $\mathbb{F}_3[x]/(x^3+x+1)$ is not a field. Here $\mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}$ is a field with p elements.
2. Decide whether or not $x^4 + 6x^3 + 9x + 3$ is irreducible in $\mathbb{Q}[x]$.
3. Factor the integral polynomial $x^5 + 2x^4 + 3x^3 + 3x + 5$ in $\mathbb{F}_2[x]$, $\mathbb{F}_3[x]$ and $\mathbb{Q}[x]$.
4. Prove that a prime number p can be written as $p = m^2 + 2n^2$ with $m, n \in \mathbb{Z}$ if and only if $x^2 + 2$ has a root in $\mathbb{F}_p$. (In fact, this is true if and only if $p = 2$ or $p \equiv 1, 3 \mod 8$, proved by Fermat.)